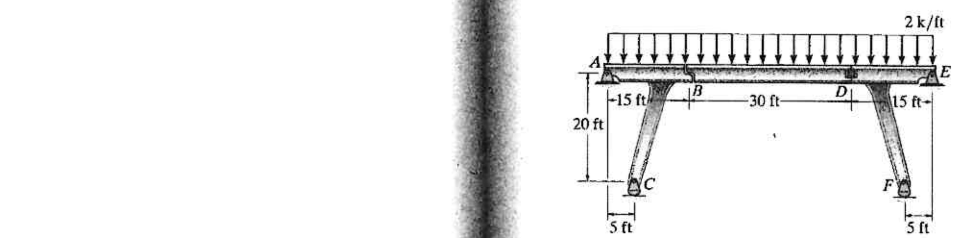
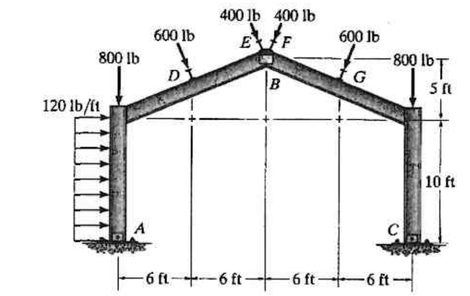
2–43. The bridge frame consists of three segments which can be considered pinned at A, D, and E, rocker supported at C and F, and roller supported at B. Determine the horizontal and vertical components of reaction at all these supports due to the loading shown.



Prob. 2–43

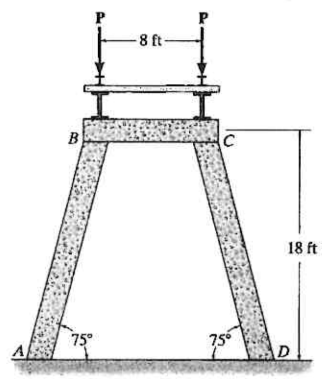
***2–44.** Determine the horizontal and vertical reactions at the connections A and C of the gable frame. Assume that A, B, and C are pin connections. The purlin loads such as D and E are applied perpendicular to the center line of each girder.



Prob. 2–44

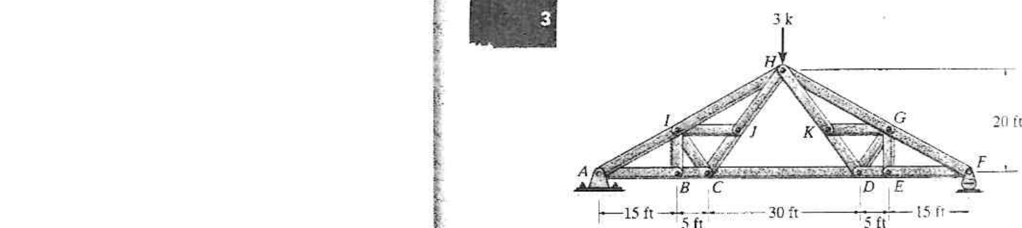
PROJECT PROBLEM

2–1P. The railroad trestle bridge shown in the photo is supported by reinforced concrete bents. Assume the two simply supported side girders, track bed, and two rails have a weight of 0.5 k/ft and the load imposed by a train is 7.2 k/ft. Each girder is 20 ft long. Apply the load over the entire bridge and determine the compressive force in the columns of each bent. For the analysis assume all joints are pin connected and neglect the weight of the bent. Are these realistic assumptions?



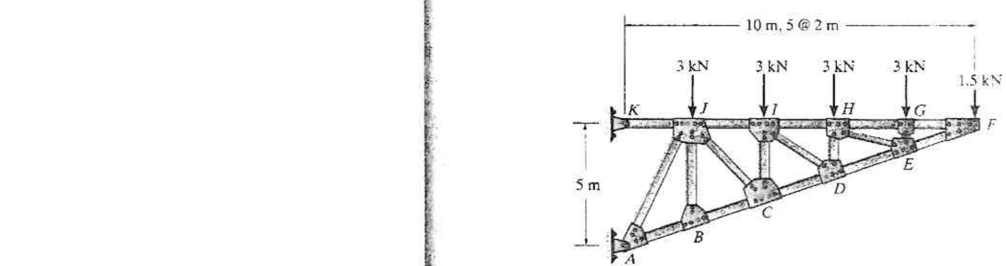
Project Prob. 2–1P

*3–28. Specify the type of compound truss and determine the forces in members *JH*, *IH*, and *CD*. State if the members are in tension or compression. Assume all members are pin connected.



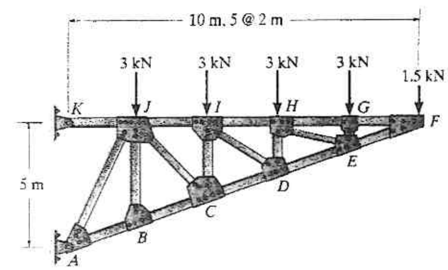
Prob. 3–28

3–29. Determine the force in members *IH*, *ID*, and *CD* of the truss. State if the members are in tension or compression. Assume all members are pin connected.



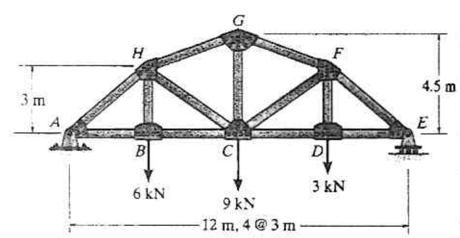
Prob. 3–29

3–30. Determine the force in members *JI*, *IC*, and *CD* of the truss. State if the members are in tension or compression. Assume all members are pin connected.



Prob. 3–30

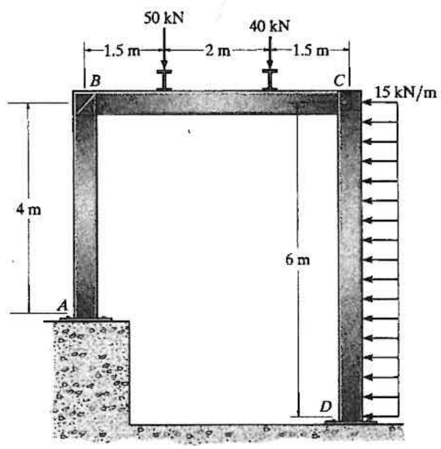
3–31. Determine the forces in members *GH*, *HC*, and *BC* of the truss. State if the members are in tension or compression. Assume all members are pin connected.



Prob. 3–31

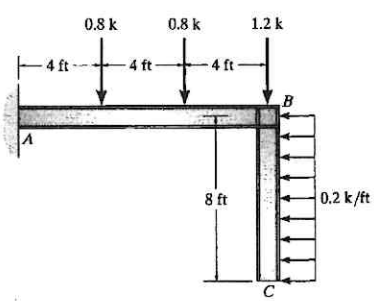
Sec. 4.4

4–38. Draw the shear and moment diagrams for each of the three members of the frame. Assume the frame is pin connected at A, C, and D and there is a fixed joint at B.



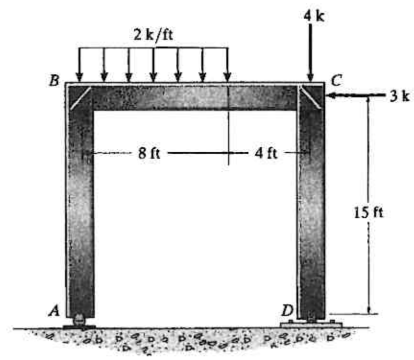
Prob. 4–38

4–39. Draw the shear and moment diagrams for each member of the frame.



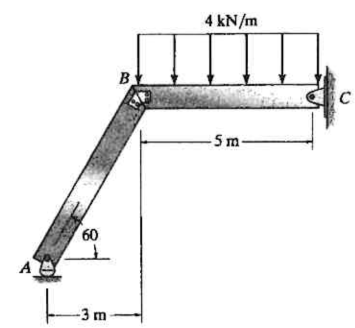
Prob. 4–39

***4–40.** Draw the shear and moment diagrams for each member of the frame. Assume A is a rocker, and D is pinned.



Prob. 4–40

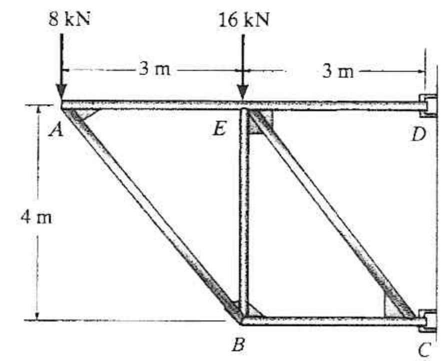
4–41. Draw the shear and moment diagrams for each member of the frame. The joint at B is fixed connected.



Prob. 4–41

9–11. Determine the vertical displacement of joint A of the truss. Each member has a cross-sectional area of $A = 300\ \text{mm}^2$. $E = 200$ GPa. Use the method of virtual work.

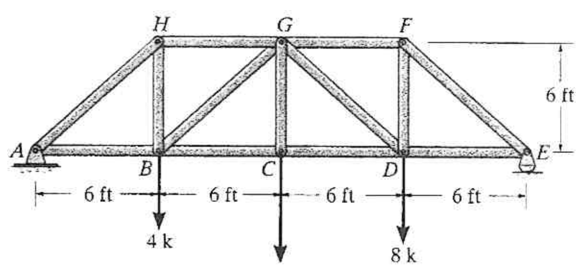
***9–12.** Solve Prob. 9–11 using Castigliano's theorem.



Probs. 9–11/12

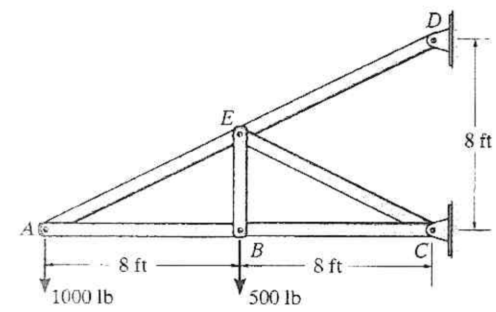
9–13. Determine the vertical displacement of joint C. Assume the members are pin connected at their end points. AE is constant. Use the method of virtual work.

9–14. Solve Prob. 9–13 using Castigliano's theorem.



9–15. Determine the vertical displacement of joint A. Assume the members are pin connected at their end points. Take $A = 2\ \text{in}^2$ and $E = 29\ (10^3)$ for each member. Use the method of virtual work.

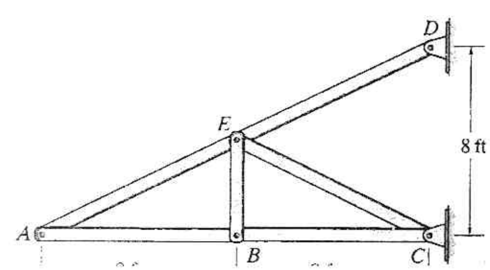
***9–16.** Solve Prob. 9–15 using Castigliano's theorem.



Probs. 9–15/16

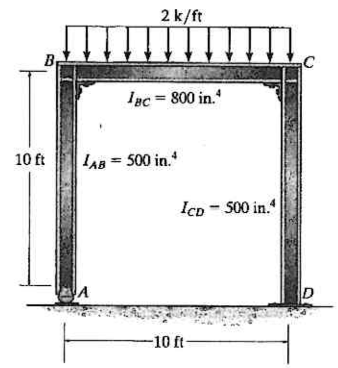
9–17. Determine the vertical displacement of joint A if members AB and BC experience a temperature increase of $\Delta T = 200°\text{F}$. Take $A = 2\ \text{in}^2$ and $E = 29(10^3)$ ksi. Also, $\alpha = 6.60\ (10^{-6})/°\text{F}$.

9–18. Determine the vertical displacement of joint A if member AE is fabricated 0.5 in. too short.



*9–44. Determine the horizontal displacement at A. Take $E = 29(10^3)$ ksi. The moment of inertia of each segment of the frame is indicated in the figure. Assume D is a pin support. Use the method of virtual work.

9–45. Solve Prob. 9–44 using Castigliano's theorem.

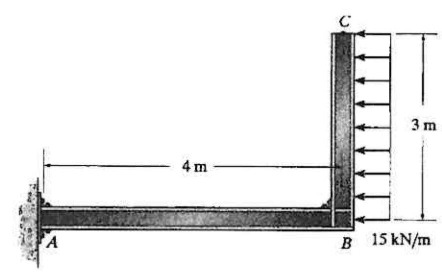


Probs. 9–44/45

9–46. The L-shaped frame is made from two fixed-connected segments. Determine the horizontal displacement of the end C. Use the method of virtual work. EI is constant.

9–47. The L-shaped frame is made from two fixed-connected segments. Determine the slope at point C. Use the method of virtual work. EI is constant.

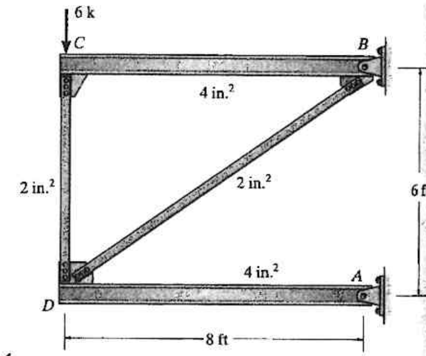
*9–48. Solve Prob. 9–47 using Castigliano's theorem.



Probs. 9–46/47/48

9–49. Determine the vertical displacement of joint D. Use the method of virtual work. Take $E = 29(10^3)$ ksi. Assume the members are pin connected at their ends.

9–50. Solve Prob. 9–49 using Castigliano's theorem.



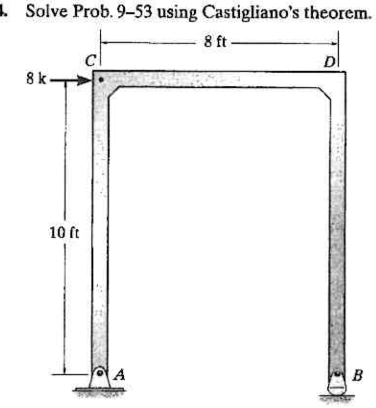
Probs. 9–49/50

9–51. Determine the horizontal displacement at C. Take $E = 29(10^3)$ ksi, $I = 150$ in⁴ for each member. Use the method of virtual work.

*9–52. Solve Prob. 9–51 using Castigliano's theorem.

9–53. Determine the horizontal displacement of the rocker at B. Take $E = 29(10^3)$ ksi, $I = 150$ in⁴ for each member. Use the method of virtual work.

9–54. Solve Prob. 9–53 using Castigliano's theorem.



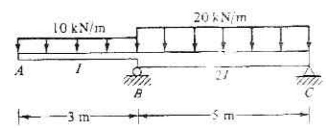
Probs. 9–51/52/53/54

9

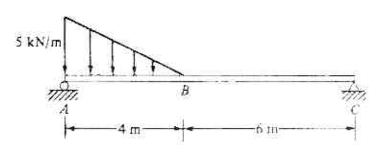
7.8 Compute the displacement of points D and B of the beam shown. Use $E = 10 \times 10^3$ kips/in.^{2} and 600 in.^{4}

7.9 Compute the displacement of point A of the beam shown. Use symmetry and values for E and I of 200.0×10^6 kN/m^2 and 10×10^{-6} m^4, respectively.

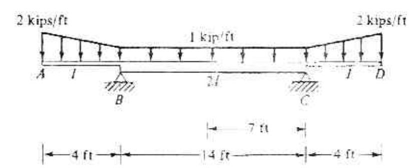
7.10 Compute the displacement of point A of the beam shown. Use $E = 200.0$ GPa and $I = 50 \times 10^{-6}$ m^4.



7.11 Compute the rotation of point C of the beam shown. Use $E = 200.0 \times 10^9$ N/m^2 and $I = 200 \times 10^{-6}$ m^4 = 200 10^{-6} m.^{4}



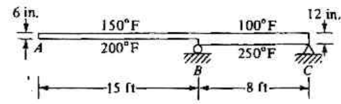
7.12 Compute the displacement of the centerline on the beam shown. Use symmetry, $E =$ 30,000 kips/in.^{2}, and $I = 250$ in.



Use the virtual work method for problems 7.31–7.49.

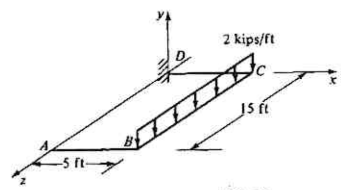
7.31–7.40 Solve Problems 7.17–7.26 by the virtual work method.

7.41–7.43 Compute the indicated displacement or rotation of the structures of Fig. P7.41–P7.43.



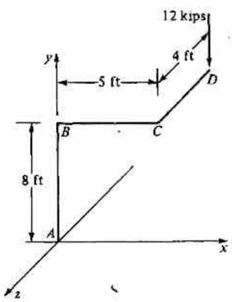
Compute Δ_A
$\alpha = 6.5 \times 10^{-6}$

P7.41



Compute Δ_A in the y direction
$I = 70{,}000$ in.^{4}
$I_x = 50{,}000$ in.^{4}
$E = 3000$ kips/in.^{2}
$G = 1500$ kips/in.^{2}

P7.42



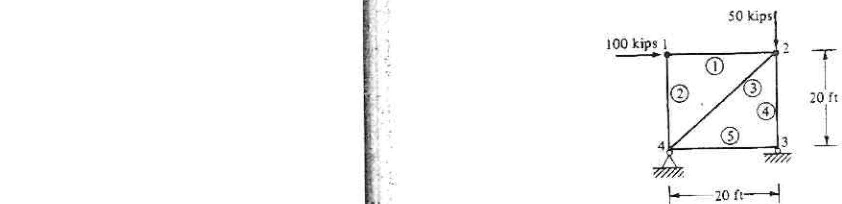
Compute Δ_D in the y direction
$I = 2000$ in.^{4}
$I_x = 6000$ in.^{4}
$E = 3000$ kips/in.^{2}
$G = 1500$ kips/in.^{2}

P7.43

7.44–7.46 For the trusses of Figs. P7.44–P7.46, use virtual work to compute the displacements indicated.

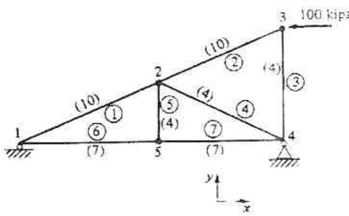
Compute Δ_x at joint 2
All areas = 5 in.^{2}
E = 30,000 kips/in.^{2}

P7.44



Compute Δ_x at joint 2
E = 30,000 kips/in.^{2}
A_1 = 5 in.^{2}
A_2 = 8 in.^{2}
A_3 = 10 in.^{2}
A_4 = 8 in.^{2}
A_5 = 5 in.^{2}

P7.45

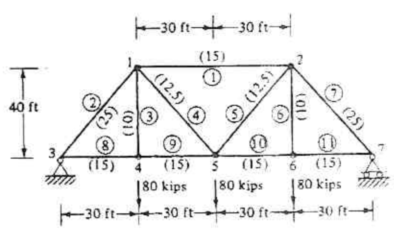


Compute Δ_x at joint 5
E = 30,000 kips/in.^{2}
Values in parentheses
are areas in inches

P7.46

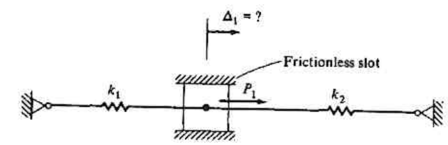
7.47 For the truss shown:

(a) Compute the vertical displacement of joint 2 due to the 80-kip loads.
(b) Determine the vertical displacement of joint 2 due to a temperature drop of 50°F of the bottom cord of the truss. Use $\alpha = 6.5 \times 10^{-6}$.
(c) Find the changes in the lengths of the bottom cord that will result in a value of zero vertical displacement of joint 2 under the loads of part a.

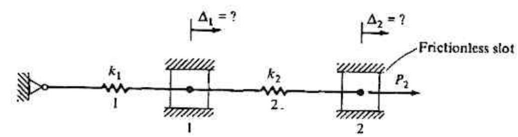


E = 30,000 kips/in.^{2}
Values in parentheses
are areas in in.^{2}.

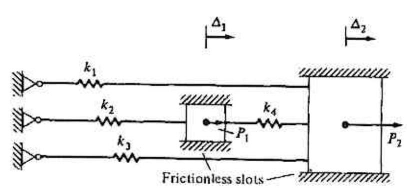
3.5 Using the stiffness method approach, compute the displacement indicated. These springs are in "parallel." What is the total stiffness of the system at point 1? Express your answers in terms of k_1 and k_2. For $k_1 = 100\text{kN/m}$, $k_2 = 50\text{kN/m}$, and $P_1 = 10$ kN, what is the displacement in centimeters? What are the forces in each spring in terms of k_1, k_2, and P_1?



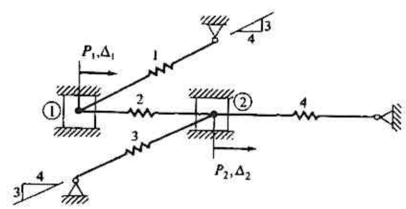
3.6 Using the stiffness method approach compute the displacements indicated. These springs are in "series." What is the total stiffness of the system at point 2? Express your answers in terms of k_1 and k_2. If $k_1 = 100\text{kN/m}$, $k_2 = 50$ kN/m, and $P = 10$ kN, what are the displacements in centimeters?



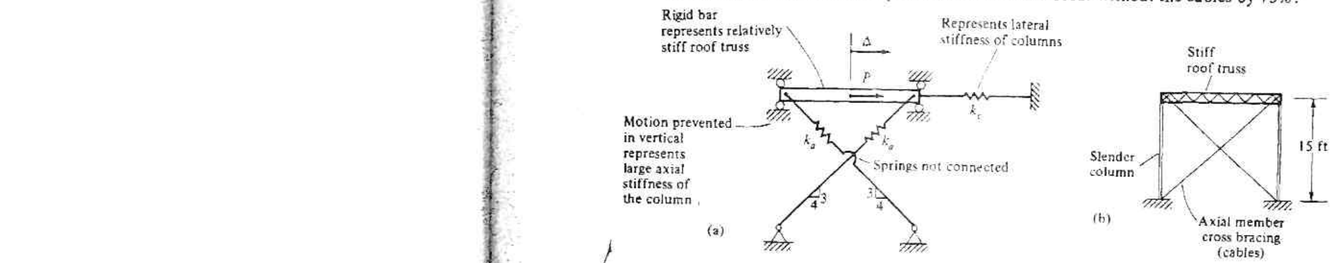
3.7 Using the stiffness method compute Δ_1 and Δ_2 for $k_1 = 20$ kips/in., $k_2 = 20$ kips/in., $k_3 = 10$ kips/in., $k_4 = 5$ kips/in., $P_1 = 50$ kips, and $P_2 = 75$ kips. What are the forces in each spring?



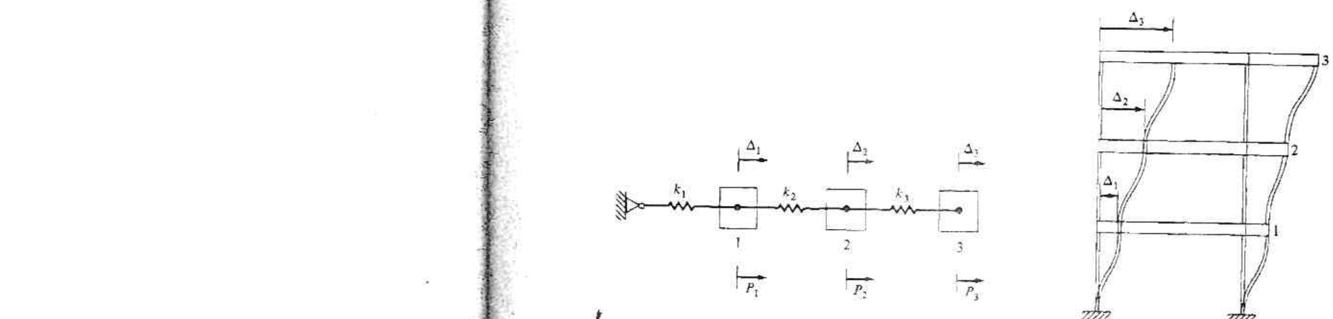
3.8 Using the stiffness method, compute the displacements and spring forces for $P_1 = 50$ kN, $P_2 = 50$ kN, $k_1 = 50$ kN/m, $k_2 = 100$ kN/m, $k_3 = 50$ kN/m, and $k_4 = 100$ kN/m.



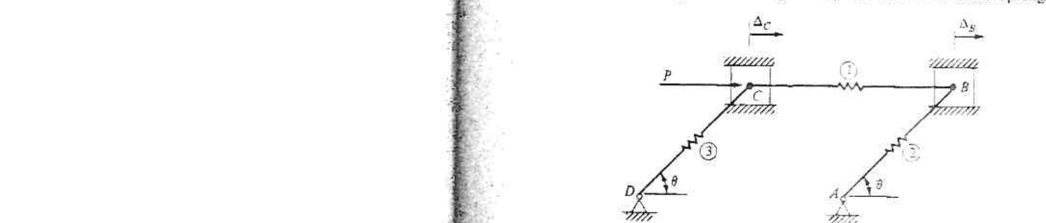
3.9 This spring system represents a reasonable model of a truss roof supported by slender columns, used to examine the effect of axial bracing. Use the stiffness method to compute the displacement in terms of k_a, k_c, and P. (*Note:* The springs k_a represent cables!) What value of k_a will reduce, the displacement that would occur without the cables by 75%?



3.10 This spring system models a three-story building with a relatively stiff floor system. In the event of an earthquake, ground motion will induce lateral inertia forces on each floor. To properly analyze this problem, a dynamic analysis must be performed. Forming the stiffness matrix is one step of the process. Construct the stiffness matrix for this structure. For $k_1 = k_2 = k_3 = 24$ kips/in. and $P_1 = 10$ kips, $P_2 = 10$ kips, and $P_3 = 7$ kips, compute the lateral displacements of the frame and the forces in the springs.



3.11 The spring model shown has two degrees of freedom Δ_C and Δ_B. Using the stiffness method with $k_1 = 2$ kips/in., $k_2 = 1$ kips/in., $k = 1$ kips/in., and $\theta = 45°$ and $P = 4$ kips, compute the displacements Δ_C and Δ_B and the forces in the springs.



3.15 Solve Example 3.4 by the stiffness method.

3.16 Resolve Example 3.4 by the flexibility method if in addition to the applied load P the support on the end of spring 1 is forced to displace downward .01 in. Also compute the final rotation of the beam.

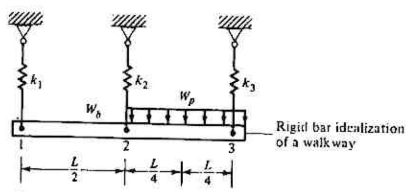
3.17 The independent displacement, or degree of freedom, for this structure is θ_0. Use the stiffness method to compute θ_0 and the spring forces.

P = 40 kN
L = 8 m
k_1 = 25 kN/m
k_2 = 50 kN/m
k_3 = 75 kN/m
k_4 = 100 kN/m

3.18 Use the flexibility method to solve for F_1, F_2, and θ. Use F_1 as the redundant.

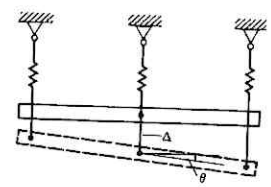
P = 24 kips
L = 15 ft
f_1 = .005 ft/kips
f_2 = .01 ft/kips

3.19 A balcony is supported by a set of cables. The balcony can be considered to behave as a rigid bar. The independent displacements may be taken to be the downward displacement of the balcony at point 2, Δ, and the rotation about 2, θ + ↺ . Use the stiffness method to find these displacements and the forces in the cables. Use L = 24 ft, w_p = 400 lb/ft, and w_b = 450 lb/ft. The stiffness of all k are the same. What value of k will limit the ***maximum*** displacement to ½ in.?



W_b = balcony weight

(a)

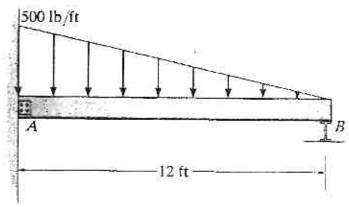


Degrees of freedom Δ, θ

(b)

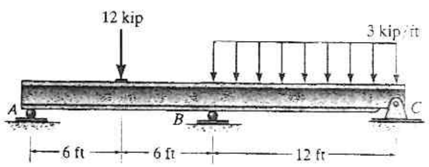
Sec. 10.1–10.4

10–1. Determine the reactions at the supports then draw the moment diagram. Assume the support at B is a roller.



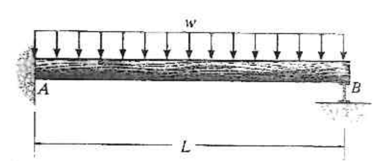
Prob. 10–1

10–2. Determine the reactions at the supports A, B, and C, then draw the shear and moment diagrams. EI is constant.



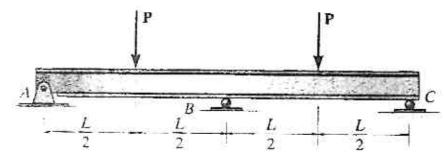
Prob. 10–2

10–3. Determine the reactions at the supports, then draw the moment diagram. Assume the support at B is a roller. EI is constant.



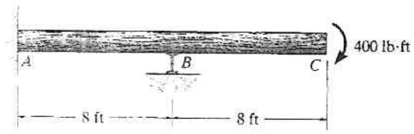
Prob. 10–3

***10–4.** Determine the reactions at the supports A, B, and C; then draw the shear and moment diagram. EI is constant.



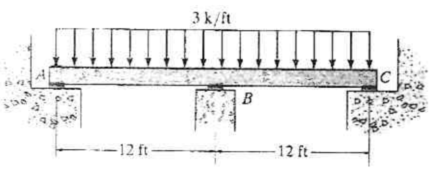
Prob. 10–4

10–5. Determine the reactions at the supports, then draw the moment diagram. Assume the support at B is a roller. EI is constant.



Prob. 10–5

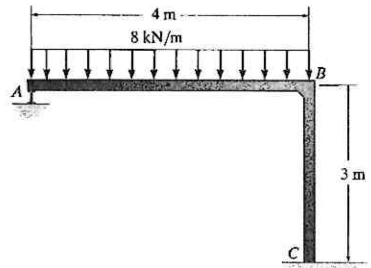
10–6. Determine the reactions at the supports, then draw the moment diagram. Assume B and C are rollers and A is pinned. The support at B settles downward 0.25 ft. Take $E = 29(10^3)$ ksi, $I = 500$ in^4.



Prob. 10–6

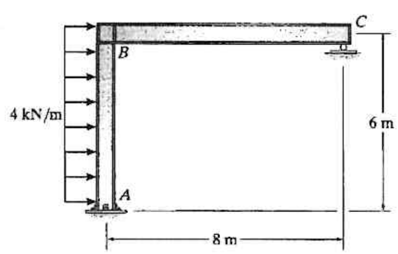
10

10–17. Determine the reactions at the supports, then draw the moment diagram for each member. EI is constant.



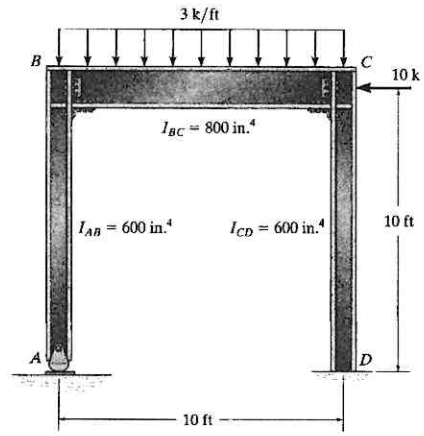
Prob. 10–17

10–18. Determine the reactions at the supports. Assume A is a fixed and the joint at B is fixed connected. EI is constant.



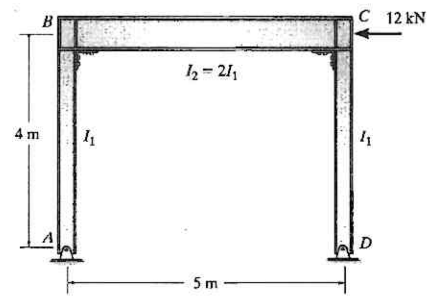
Prob. 10–18

10–19. Determine the reactions at the supports. E is constant.



Prob. 10–19

***10–20.** Determine the reactions at the supports, then draw the moment diagram for each member. EI is constant.



Prob. 10–20

10–21. D
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10–22. De
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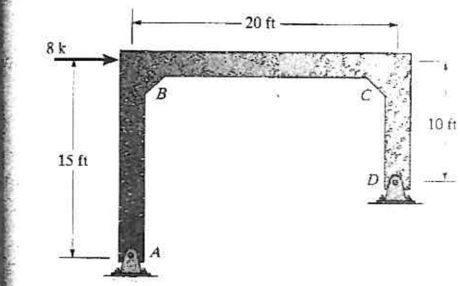
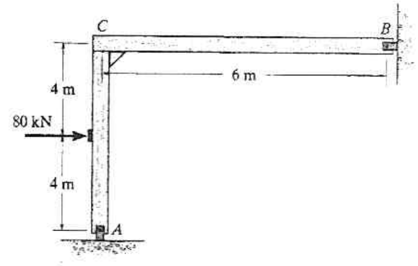
are pins and

10–21. Determine the reactions at the supports. Assume A and D are pins. EI is constant.



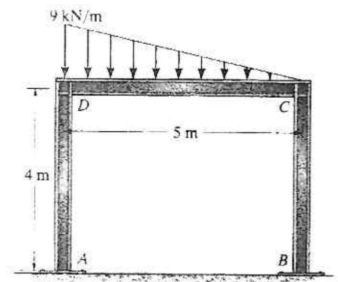
Prob. 10–21

10–22. Determine the reactions at the supports, then draw the moment diagrams for each member. Assume A and B are pins and the joint at C is fixed connected. EI is constant.



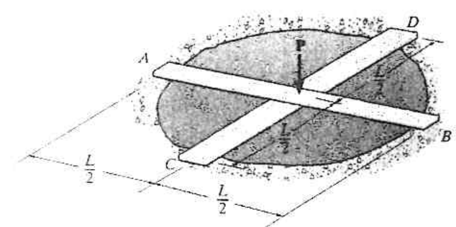
Prob. 10–22

10–23. Determine the reactions at the supports. Assume A and B are pins. EI is constant.



Prob. 10–23

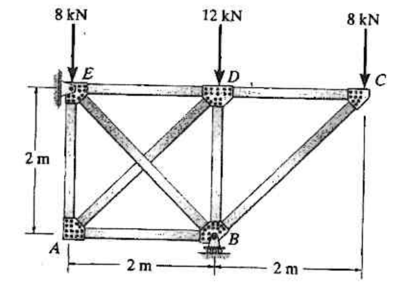
***10–24.** Two boards each having the same EI and length L are crossed perpendicular to each other as shown. Determine the vertical reactions at the supports. Assume the boards just touch each other before the load **P** is applied.



Prob. 10–24

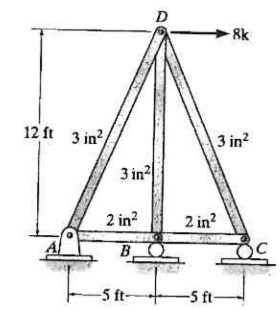
10

10–29. Determine the force in member AD of the pin-connected truss. AE is constant.



Prob. 10–29

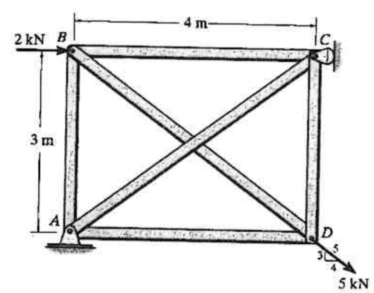
***10–32.** Determine the force in each member of the truss. AE is constant.



Prob. 10–32

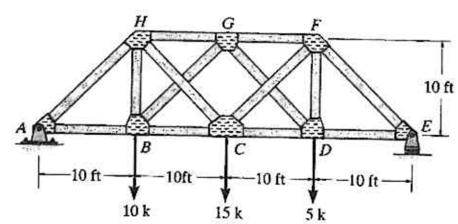
10–30. Determine the force in member BD. AE is constant.

10–31. Determine the force in member BC. AE is constant.



Probs. 10–30/31

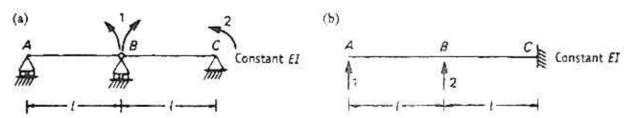
10–33. Determine the force in member GB of the truss. AE is constant.



Prob. 10–33

analysis, with attention directed to the procedure of the force method, rather than to the methods of computation of displacements. These will be treated in subsequent chapters. Additional problems on the application of the force method can be found at the end of Chapter 8, which requires calculation of displacements by method of virtual work.

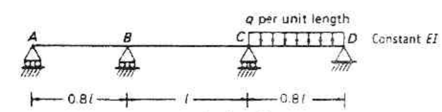
4.1 Write the flexibility matrix corresponding to coordinates 1 and 2 for the structures shown below.



Prob. 4.1

4.2 Use the flexibility matrices derived in Prob. 4.1 to find two sets of redundant forces in two alternative solutions for the continuous beam of Example 4.1.

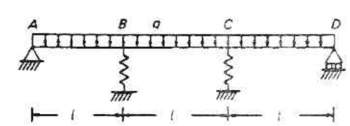
4.3 Use the force method to find the bending moment at the intermediate supports of the continuous beam shown in the figure.



Prob. 4.3

4.4 Obtain the bending moment diagram for the beam of Prob. 4.3 on the assumption that support B settles vertically a distance $l/1200$. (No distributed load acts in this case.)

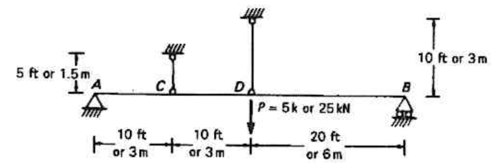
4.5 Use the force method to find the bending moments at the supports of a continuous beam on elastic (spring) supports. The beam has a constant flexural rigidity EI, and the stiffness of the elastic supports is $K = 20EI/l^3$.



Prob. 4.5

4.6 Use the force method to find the forces in the three springs A, B, and C in the system shown. The beams DE and FG have a constant flexural rigidity EI, and the

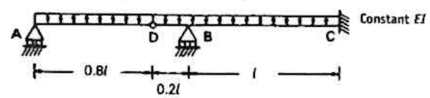
For the beam $I = 40$ in.^{4}, for the cables $a = 0.15$ in.^{2}; the modulus of elasticity for both is $E = 30 \times 10^3$ ksi, and the coefficient of thermal expansion for steel is 6.5×10^{-6} per degree Fahrenheit.



Prob. 4.8 (imperial units) or Prob. 4.9 (SI units)

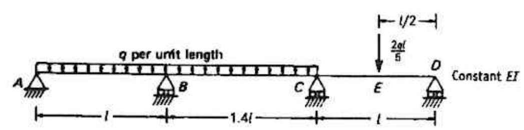
4.9 SI units. A steel beam AB is supported by two steel cables at C and D. Using the force method, find the tension in the cables and the bending moment at D due to a load $P = 25$ kN and a drop of temperature of 20 degrees Celsius in the two cables. For the beam $I = 16 \times 10^6$ mm^4, for the cables $a = 100$ mm^2; the modulus of elasticity for both is $E = 200$ GN/m^2, and the coefficient of thermal expansion for steel is 1×10^{-5} per degree Celsius.

4.10 Using the equation of three moments, find the bending moment diagram for the beam shown.



Prob. 4.10

4.11 Using the equation of three moments, obtain the bending moment and shearing force diagrams for the continuous beam shown.

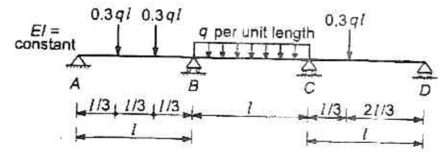


Prob. 4.11

4.12 For the beam in Fig. 4.1a, but without the uniform load, find the reactions at the supports and the bending moment diagram due to a rise in temperature varying linearly over the beam depth h. The temperature rise in degrees at top and bottom fibers is T_t and T_b, respectively. The coefficient of thermal expansion is α per degree.

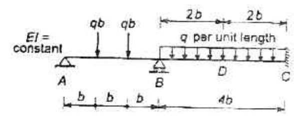
shearing force diagrams due to a unit downward settlement of support B. (The main answers for this problem are included in Table E-3, Appendix E. Note that the presence of the overhang DA has no effect.)

4.18 For the continuous beam shown, determine: (a) the bending moment diagram, (b) the reaction at B and (c) the deflection at the center of BC.



Prob. 4.18

4.19 For the continuous beam shown, determine the bending moment diagram, the reaction at B and the deflection at D due to the given loads. What is the reaction at B due to the downward settlement δ at support B?



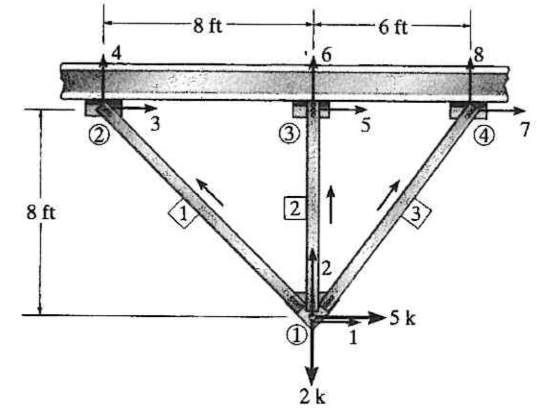
Prob. 4.19

4.20 Consider the beam in Fig. 4.11. Due to uniform load q /unit length covering the whole length, combined with a uniform live load $p = q$ per unit length, find: $M_{B\max}-$, $M_{E\max}+$ and $R_{C\max}+$. E is at the middle of AB; EI is constant.

Problems

14–1. Determine the stiffness matrix **K** for the truss. AE is constant.

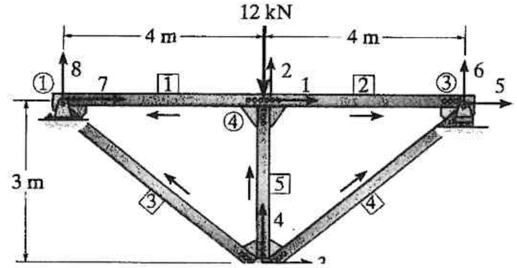
14–2. Determine the force in each member of the truss in Prob. 14–1. AE is constant.



Probs. 14–1/2

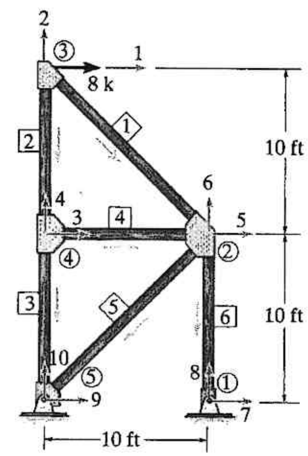
14–3. Determine the stiffness matrix **K** for the truss. Take $A = 0.0015\ m^2$ and $E = 200$ GPa for each member.

***14–4.** Determine the vertical deflection at joint ② and the force in member 4 of the truss in Prob. 14–3. Take $A = 0.0015\ m^2$ and $E = 200$ GPa for each member.



14–5. Determine the stiffness matrix **K** for the truss. AE is constant.

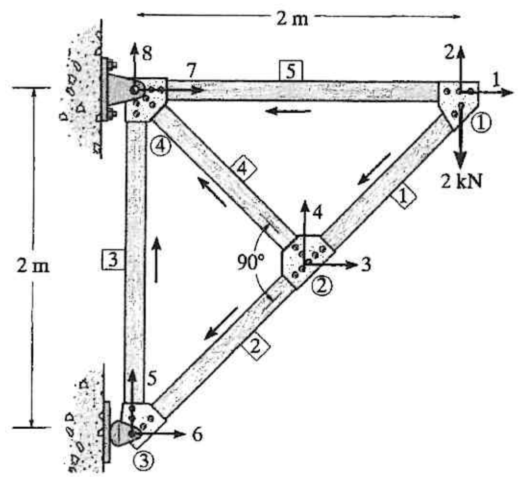
14–6. Determine the horizontal displacement of joint ③ and the force in member 1. AE is constant.



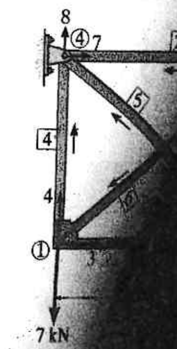
Probs. 14–5/6

14–7. Determine the stiffness matrix **K** for the truss. AE is constant.

***14–8.** Determine the force in members 1 and 5 of the truss in Prob. 14–7. AE is constant.



14–9. Determine the stiffness
Take $A = 0.0015\ m^2$ and $E = 200$



14–10. Determ
Prob. 14–9. Take
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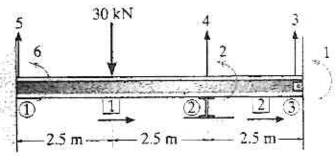
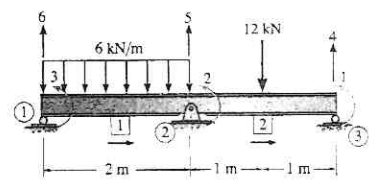
15–1. Determine the internal moment in the beam at ① and ②. Assume ② is a roller and ③ is a pin. EI is constant.



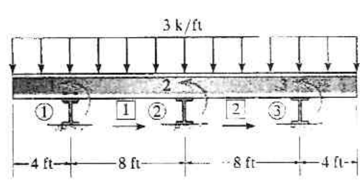
Prob. 15–1

15–2. Determine the reactions at the supports. EI is constant.



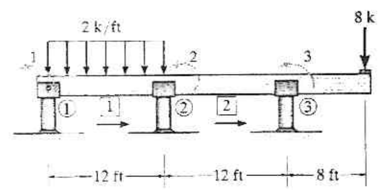
Prob. 15–2

15–3. Determine the reactions at the supports ①, ②, and ③. Assume ① is pinned, ② and ③ are rollers. EI is constant.



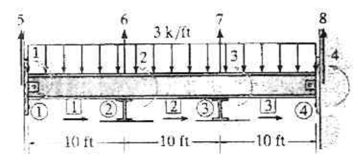
Prob. 15–3

*15–4. Determine the reactions at the supports ①, ②, and ③. Assume ① is pinned ② and ③ are rollers. EI is constant.



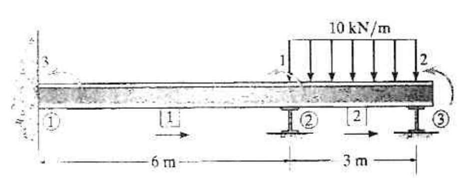
Prob. 15–4

15–5. Determine the moments at ② and ③. Assume ② and ③ are rollers and ① and ④ are pins. EI is constant.



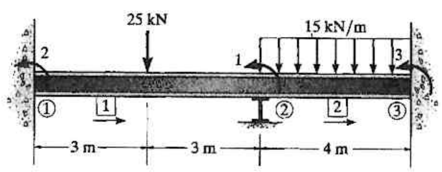
Prob. 15–5

15–6. Determine the internal moment in the beam at ① and ②. EI is constant. Assume ② and ③ are rollers.



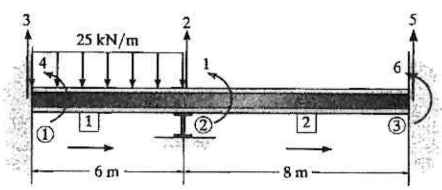
Prob. 15–6

15–7. Determine the moments at the supports ① and ③. EI is constant. Assume joint ② is a roller.



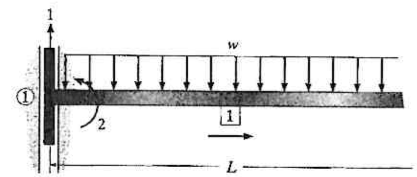
Prob. 15–7

***15–8.** Determine the moments at the supports. Assume ② is a roller. EI is constant.



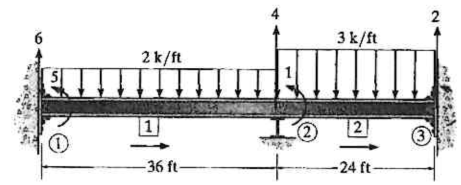
Prob. 15–8

15–9. Determine the reactions at the supports slider at ①. EI is constant.



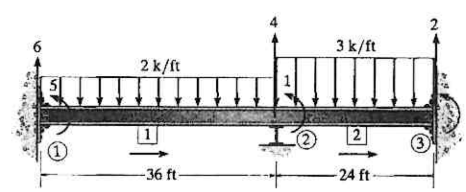
Prob. 15–9

15–10. Determine the moments at ① and ③. Assume a roller and ① and ③ are fixed. Also, here EI is constan



Prob. 15–10

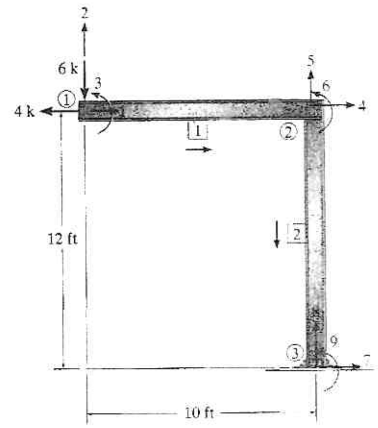
15–11. Determine the moments at ① and ③ if the support ② settles 0.1 ft. Assume ② is a roller and ① and ③ are fixed. $EI = 9500\ \text{k} \cdot \text{ft}^2$.



Prob. 15–11

16–7. Determine the structure stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 650$ in^4, $A = 20$ in^2 for each member.

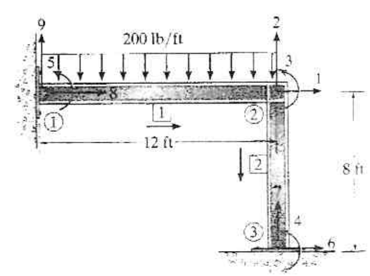
*__16–8.__ Determine the components of displacement at ①. Take $E = 29(10^3)$ ksi, $I = 650$ in^4, $A = 20$ in^2 for each member.



Probs. 16–7/8

16–9. Determine the structure stiffness matrix **K** for the frame. Assume ① and ③ are pins. Take $E = 29(10^3)$ ksi, $I = 600$ in^4, $A = 10$ in^2 for each member.

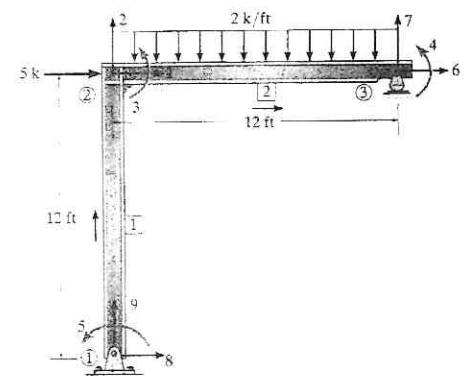
16–10. Determine the internal loadings at the ends of each member. Assume ① and ③ are pins. Take $E = 29(10^3)$ ksi, $I = 600$ in^4, $A = 10$ in^2 for each member.



Probs. 16–9/10

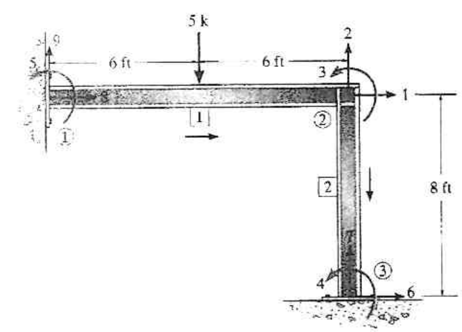
16–11. Determine the structure stiffness matrix **K**, for each member of the frame. Take $E = 29(10^3)$ ksi, $I = 700$ in^4, $A = 30$ in^2 for each member. Joint ① is pin connected.

*__16–12.__ Determine the support reactions at ① and ③. Take $E = 29(10^3)$ ksi, $I = 700$ in^4, $A = 30$ in^2 for each member. Joint ① is pin connected.



Probs. 16–11/12

16–13. Determine the structure stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 600$ in^4, $A = 10$ in^2 for each member. Assume joints ① and ③ are pinned; joint ② is fixed.



Prob. 16–13

cedure as in Section 11.10, we obtain for the beam in Fig. 14.4b, with a hinged end A, the end-moment at B.

$$M_{AB}^{①} = M_{BA} - CM_{AB} \tag{14.36}$$

where $C = t/S$. This equation is valid regardless of whether the axial force is compressive or tensile.

For the beam in Fig. 14.4c, with translation of end A in the transverse direction allowed but rotation prevented, the end-moments are

$$M_{AB}^{②} = M_{AB} + F_A l \frac{(S+t)}{2(S+t) \pm Pl} \tag{14.37}$$

$$M_{BA}^{②} = M_{BA} + F_A l \frac{(S+t)}{2(S+t) \pm Pl} \tag{14.38}$$

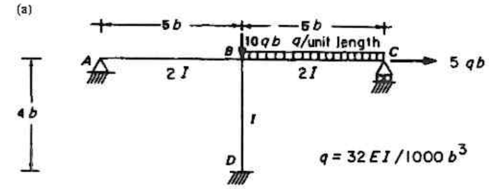
where M_{AB}, M_{BA}, and F_A are the end-moments and reaction for the same beam but with the end condition of Fig. 14.4a; the positive directions of these forces are indicated in the figure. The sign of the term Pl[that is equal to $u^2(EI/l)$] in the denominator in the last two equations is plus when the axial force is tensile and minus when compressive.

Example 14.1 Plane frame without joint translations

Find the end-moments for the members in the frame of Fig. 14.5a, taking into account the beam–column effect.

We assume approximate values of the axial forces in the members: $P_{AB} = P_{BC} = 5qb$ tensile, and $P_{BD} = 12.5qb$ compressive. The corresponding values of $u = l\sqrt{P/(EI)}$ are: $u_{AB} = u_{BC} = 1.41$, and $u_{BD} = 2.53$.

(a)



(b)

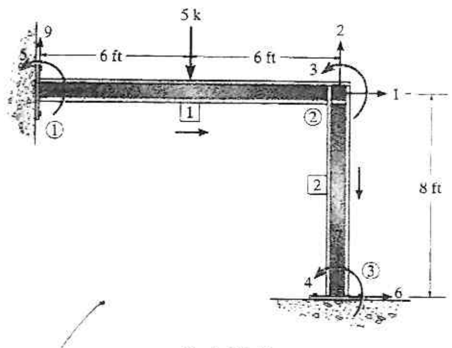
End	BA	BD	BC
DFs	0.39	0.22	0.39
FEMs			-295
Distribution	115	65	115
Final moments	115	65	-180

Multiplier: $qb^2/100$

COF $C_{BD} = 0.74$ $M_{DB} = 0.74 \times 65 = 48$

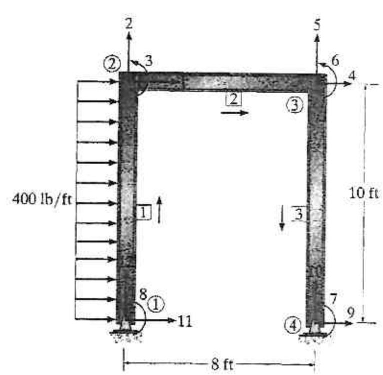
Fig. 14.5 Analysis of the frame of Example 14.1 by moment distribution taking into account the beam – column effect. (a) Frame properties and loading. (b) Moment distribution.

16–14. Determine the rotation at ① and ③ and the support reactions. Take $E = 29(10^3)$ ksi, $I = 600$ in^4, $A = 10$ in^2 for each member. Assume joints ① and ③ are pinned; joint ② is fixed.



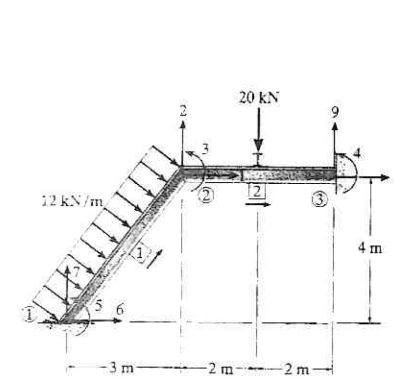
Prob. 16–14

16–15. Determine the reactions at the supports ① and ④. Joints ① and ④ are pin connected and ② and ③ are fixed connected. Take $E = 29(10^3)$ ksi, $I = 700$ in^4, $A = 15$ in^2 for each member.



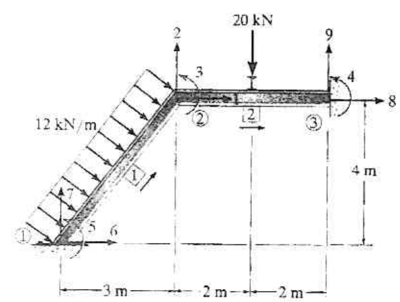
Prob. 16–15

***16–16.** Determine the structure stiffness matrix **K** for the two-member frame. Take $E = 200$ GPa, $I = 350(10^6)$ mm^4, $A = 20(10^3)$ mm^2 for each member. Joints 1 and 3 are pinned and joint 2 is fixed.



Prob. 16–16

16–17. Determine the support reactions at ① and ③ in. Take $E = 200$ GPa, $I = 350(10^6)$ mm^4, $A = 20(10^3)$ mm^2 for each member. Joints ① and ③ are pinned and joint ② is fixed.



Prob. 16–17

state and resist bending only, while the middle portion is in an elastic state and resists shear.

2. The shearing stress in the (rectangular) center portion is calculated by ordinary elastic equations.

The resistance of the section is assumed to be exhausted when the shearing stress in the center portion reaches yield under pure shear, which, according to von Mises-Hencky yield criterion, equals $\sigma_y/\sqrt{3}$. With these assumptions, the following equation can be derived:[3]

$$\frac{M_{ps}}{M_p} = \frac{8bc^2}{9\alpha Z}\left(\sqrt{1 + \frac{9}{4}\frac{\alpha Z}{bc^2}} - 1\right) \qquad (19.23)$$

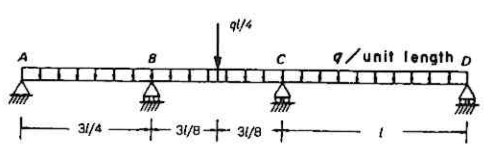
where b is the width of the rectangular section or of the web of an I-section, Z is the section modulus, α is as defined at the end of Section 19.2, and c is the shear span (= bending moment/shearing force).

19.10 General

This chapter is no more than an introduction to the structural analysis required in the plastic design of steel structures. For safe and efficient use of plastic design of continuous framed structures, it is important to understand the restrictions and limitations of this design method, such as the effect of repeated loading, instability, and also to be able to estimate deflections at working and ultimate loads.[4]

Problems

19.1 Find the required plastic moment resistance of the cross section(s) for the beam in the figure, which is to be designed to carry the given loads with a load factor of 1.7. Assume that the beam has: (a) a constant cross section, (b) two different cross sections one from A to C, and the other from C to D.

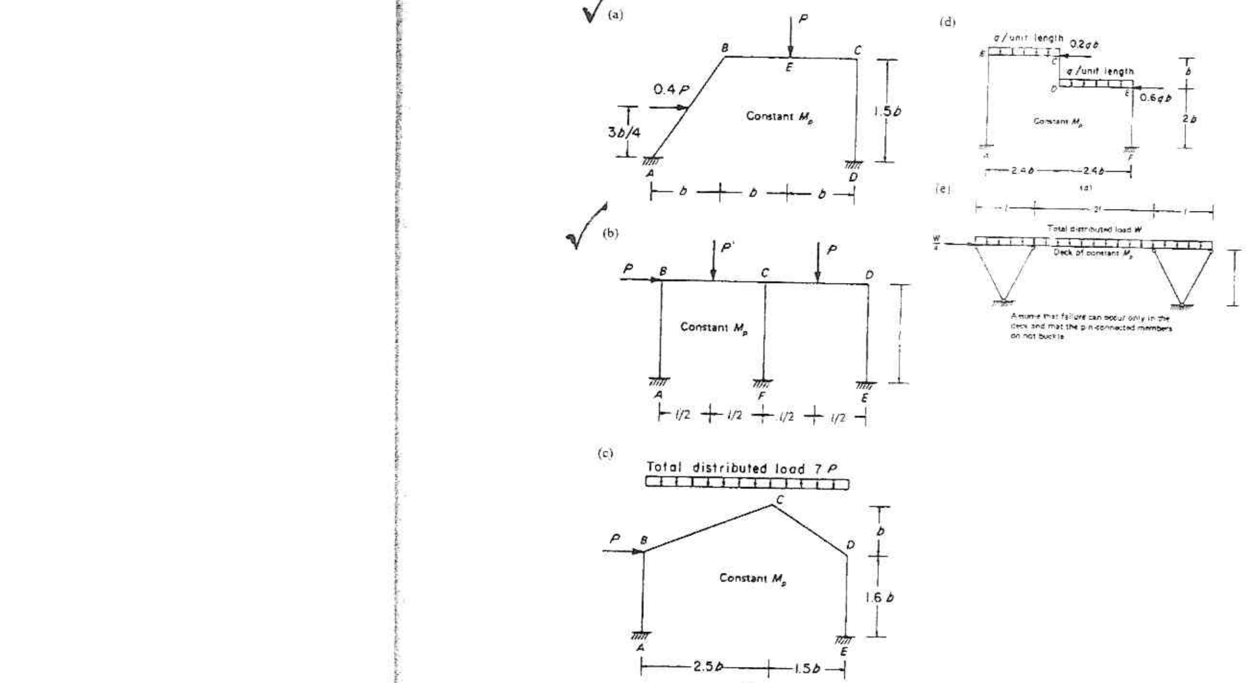


Prob. 19.1

[3] See pp. 35–40 of the reference in footnote 2 in this chapter for proof and limitations of this equation, and also for the combined effect of shear and axial force.

[4] See, for instance, Neal, B. G., *The Plastic Methods of Structural Analysis*, 2nd ed., Chapman and Hall, London, 1963. A list of other references can be found at the end of the manual referred to in footnote 2 in this chapter.

19.2 Determine the fully plastic moment for the frames shown, with the collapse loads indicated. Ignore the effects of shear and axial forces.



(c)
Prob. 19.2

19.3 What is the value of M_p for the frame in Prob. 19.2b, if the axial force effect (excluding buckling) is taken into account. Assume that the frame has a constant rectangular section $b \times d$, with $d = l/15$.

19.4 What is the value of M_p for the beam in Prob. 19.1, if the shear effect is taken into account. Assume that the beam has a constant rectangular section $b \times d$, with $d = l/30$.

6.14. A circular bar with central angle 80° is clamped at point A and free at point B. The bar is subjected to horizontal force P at the free end B (Fig. P6.14). The area A of cross section of the bar and moment of inertia I are constant. Calculate the horizontal displacement Δ_B at point B. All the three terms of Maxwell–Mohr's integral should be used. Estimate each term for the following data: cross section is rectangular ($h = 2b$), $h/R = 0.1$, the shear modulus $G = E/2(1 + \nu)$, the Poisson's coefficient $\nu = 0.25$ and coefficient $\mu = 1.2$.

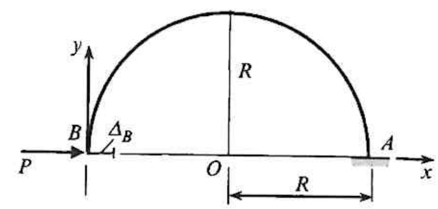


Fig. P6.14

$$\text{Ans. } \Delta_B = \frac{PR^3}{EI}\frac{\pi}{2} + \frac{PR}{EA}\frac{\pi}{2} + \frac{\mu PR}{GA}\frac{\pi}{2}.$$

6.15. Find the horizontal and angular displacement at support C of the frame shown in Fig. P6.15, when the indoor temperature rises by 10°C and outdoor temperature rises by 30°C and 20°C for elements AB and BC, respectively. The height and temperature coefficients of the elements BC and AB are b_1, α_1 and b_2, α_2, respectively.

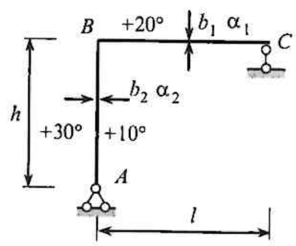


Fig. P6.15

$$\text{Ans. } \Delta_{Ct} = 5\alpha_1 l\left(\frac{h}{b_1} + 3\right) + 10\alpha_2\frac{h^2}{b_2}\left(1 + 2\frac{b_2}{l}\right)\ (\rightarrow),$$

$$\theta_C = 5\alpha_1\frac{l}{b_1} + 20\alpha_2\frac{h}{l}\quad \text{(clockwise).}$$

6.16. Design diagram of the truss is shown in Fig. P6.16. Temperature of the top chord of the truss decreases by 30°C, and of the bottom chord increases by +45°C; the temperature of diagonals and vertical elements remain constant. The coefficient of thermal expansion of material is α. Compute the

important advantage of influence lines is as follows: Influence lines for primary unknown and any factor (reaction, bending moment, the angle of rotation, etc.) for statically indeterminate structure allows calculating not only these unknown and corresponding factor, but also finding a *distribution* of internal forces for any types of fixed loads. It may be done combining the fixed and moving load approaches. It is a principal feature of influence lines for statically indeterminate structures. Ability of an engineer to apply both methods separately and together increases his opportunity of analysis and allows performing in-depth qualities and quantities investigation of structural behavior.

Problems

10.1. Continuous beam is shown in Fig. P10.1. Trace the models of the following influence lines:

(a) Reaction of all supports
(b) Bending moments at all supports and at sections n, and k
(c) Bending moment for sections which coincide with the left and right foci points (F^{L}, F^{R})
(d) Shear for sections$n$ and k and for sections 1 and 4
(e) Shear for sections which are placed infinitely close left and right to support 2

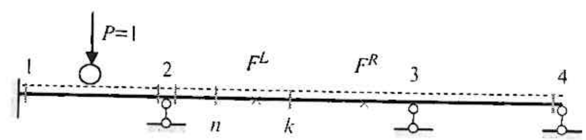


Fig. P10.1

10.2. Design diagram of a structure is shown in Fig. P10.2. Trace the models of the following influence lines:

(a) Reaction of all supports
(b) Bending moments at all supports and at section k
(c) Shear for section k and Shear for sections 1 and 4
(d) Shear for sections which are placed infinitely close to left and right to support 2

For problems (a)–(d) take into account indirect load application.

(e) Will it be changed according to the shape of the influence line models in case of nonuniform continuous beams?
(f) Is it possible to change the sign of influence line as the result of the changing of the stiffness of the structure?

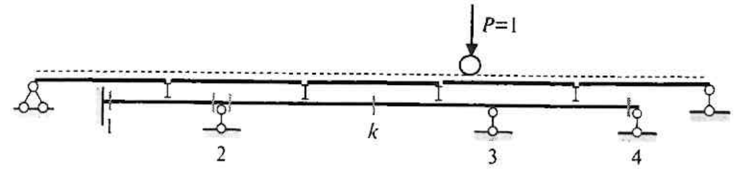


Fig. P10.2

10.3. Uniform clamped–pinned beam is shown in Fig. P10.3.

(a) Construct the influence lines for reactions R_A and R_B, moment at clamped support A, bending moment, and shear at section $k(u = 0.4)$;
(b) Construct the bending moment diagram if force $P = 10\,\text{kN}$ is placed at point $u = 0.6$. Use the above constructed influence lines.

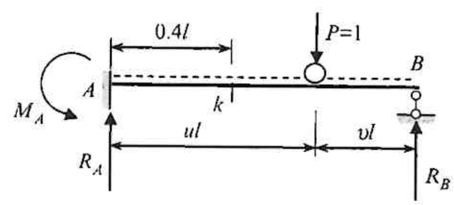


Fig. P10.3

Ans. (b) $R_B = 4.32\,\text{kN}$, $M_A = 1.68l\,\text{kNm}$

10.4. Design diagram of a frame is presented in Fig. P10.4. The relative flexural stiffness are shown in the circle; $a = b = 0.5l$.

1. Construct the influence line for horizontal reaction H at support A and bending moment at section k
2. Construct the bending moment diagram if force $P = 100\,\text{kN}$ is placed at point $x/l = 0.75$. Use the influence line for primary unknown.

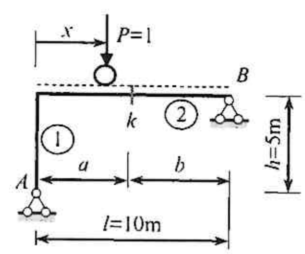


Fig. P10.4

Ans. $H = 11.718\,\text{kN}$; $R_A = 30.859\,\text{kN}$, $M_k = 95.7\,\text{kN m}$