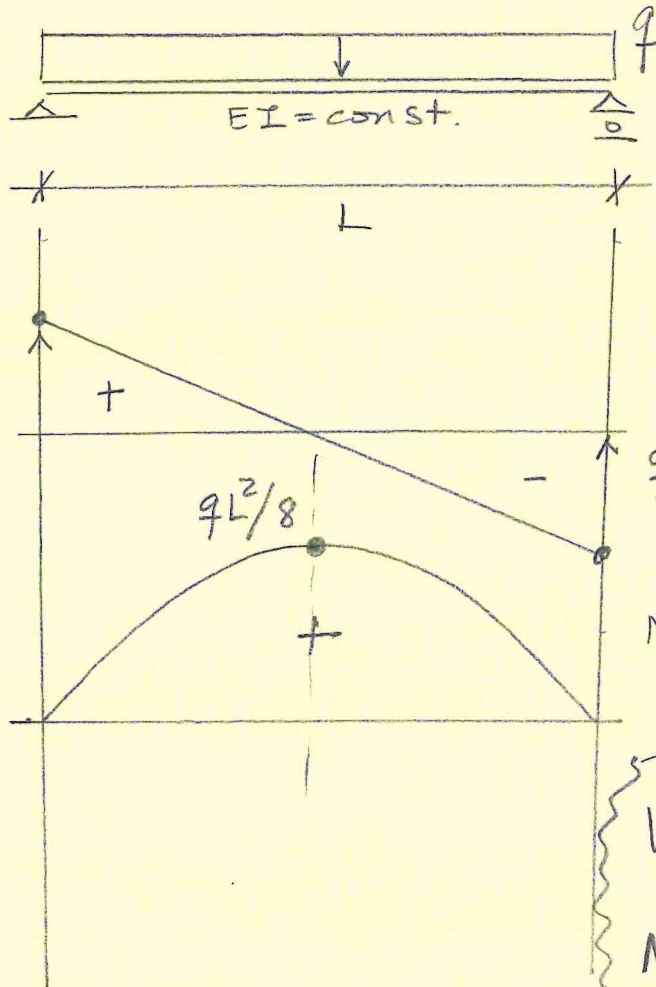


$$V(x) = \frac{qL}{2} - qx \quad (1)$$

$$M(x) = \frac{qL}{2}x - \frac{qx^2}{2} \quad (2)$$

$$V_{MAX} = \frac{qL}{2}$$

$$M_{MAX} = \frac{qL^2}{8}$$

METHOD # 1 - DOUBLE INTEGRATION

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{dy}{dx} = \int \frac{M}{EI}dx = \int \left(q\frac{Lx}{2} - q\frac{x^2}{2}\right)\frac{dx}{EI} =$$

$$EI\frac{dy}{dx} = q\frac{Lx^2}{4} - q\frac{x^3}{6} + C_1$$

$$EIy = \int\left(\frac{dy}{dx}\right)dx = \int q\frac{Lx^2}{4} - q\frac{x^3}{6} + C_1$$

$$EIy = q\frac{Lx^3}{12} - q\frac{x^4}{24} + C_1x + C_2$$

TO FIND INTEGRATION CONSTANTS EXAMINE BOUNDARY CONDITIONS

$$y(x=0) = 0 \Rightarrow \frac{qL(0)^3}{12} - \frac{q(0)^4}{24} + C_1(0) + C_2 = 0$$

THIS LEADS TO $C_2 = 0$

$$y(x=L) = 0 \Rightarrow \frac{qL^4}{12} - \frac{qL^4}{24} + C_1 L = 0$$

THIS LEADS TO $C_1 L = -\frac{qL^4}{12} + \frac{qL^4}{24}$

$$C_1 = -\frac{qL^3}{24}$$

$$EI\,y = \frac{qLx^3}{12} - \frac{qx^4}{24} - \frac{qL^3}{24}x \Rightarrow \text{deformation} \quad (3)$$

$$EI\frac{dy}{dx} = \frac{qLx^2}{4} - \frac{qx^3}{6} - \frac{qL^3}{24} \Rightarrow \text{slope} \quad (4)$$

when slope = 0 ⇒ deformation is maximum

$$EI\frac{dy}{dx} = 0 = \left(\frac{qL}{4} - \frac{qx}{6}\right)x^2 - \frac{qL^3}{24} = 0$$

try ⇒ $x = \frac{L}{2}$

$$\left(\frac{qL}{4} - \frac{qL}{12}\right)\frac{L^2}{4} - \frac{qL^3}{24}$$

$$\frac{\cancel{2}qL}{12}\cdot\frac{L^2}{\cancel{4}_{2}} - \frac{qL^3}{24} = 0 \quad \text{verified!}$$

max. def. occurs at midspan...

DETERMINE VALUE OF MAX. DEFORMATION (Eq. 3 page 2)

$$EI\,y\left(x=\frac{L}{2}\right) = \frac{qL\cdot\left(\frac{L}{2}\right)^3}{12} - \frac{q\left(\frac{L}{2}\right)^4}{24} - \frac{qL^3}{24}\cdot\left(\frac{L}{2}\right) = y_{max}$$

$$EI\,y_{MAX} = \frac{qL^4}{96} - \frac{qL^4}{384} - \frac{qL^4}{48}$$

$$\boxed{y_{MAX} = -\frac{1}{EI}\cdot\frac{5qL^4}{384}} \quad (5)$$

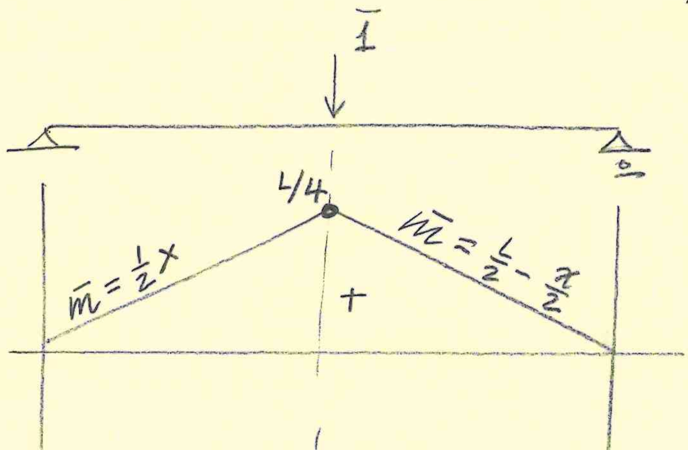
METHOD # 2 - VIRTUAL WORK

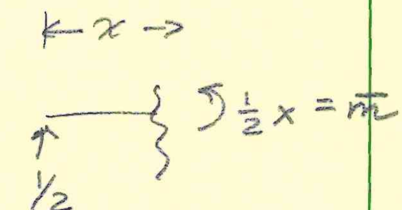
APPLY UNIT VIRTUAL LOAD AT LOC. & DIRECTION WHERE DEF. IS WANTED AND EVALUATE:

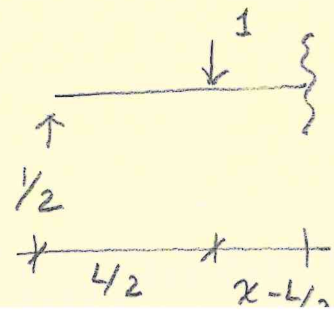
$$\delta = \int \frac{\bar{m}M}{EI}\,dx$$

$\bar{m} \Rightarrow$ moment eq. due to unit virt. load

$M \Rightarrow$ moment eq. due to real loads







$$\bar{m} = \frac{1}{2}x - 1\left(x - \frac{L}{2}\right)$$
$$= \frac{L}{2} - \frac{x}{2}$$

$$\frac{1}{EI}\int_0^{L/2}\left(\frac{qLx}{2}-\frac{qx^2}{2}\right)\left(\frac{1}{2}x\right)dx \; +$$

NOTE: We need 2 separate integrals because $\bar{m}$ has 2 diff. equations.

$$\int_{L/2}^{L}\left(\frac{qLx}{2}-\frac{qx^2}{2}\right)\left(\frac{L}{2}-\frac{x}{2}\right)dx = \delta$$

$$EI\delta = \frac{qL}{4}\frac{x^3}{3}-\frac{q}{4}\frac{x^4}{4}\Bigg]_0^{L/2} + \frac{qL^2}{4}\frac{x^2}{2}-\frac{qL}{4}\cdot\frac{x^3}{3}$$

$$-\frac{qL}{4}\cdot\frac{x^3}{3}+\frac{q}{4}\frac{x^4}{4}\Bigg]_{L/2}^{L}$$

$$EI\delta = \left(\frac{qL^4}{96}-\frac{qL^4}{256}\right)+\left(\frac{qL^4}{8}-\frac{qL^4}{12}-\frac{qL^4}{12}+\frac{qL^4}{16}\right.$$

$$\left.-\frac{qL^4}{32}+\frac{qL^4}{96}+\frac{qL^4}{96}-\frac{qL^4}{256}\right)$$

$$\delta = \left(\frac{8-3+96-64-64+48-24+8+8-3}{768}\right)\frac{qL^4}{EI}$$

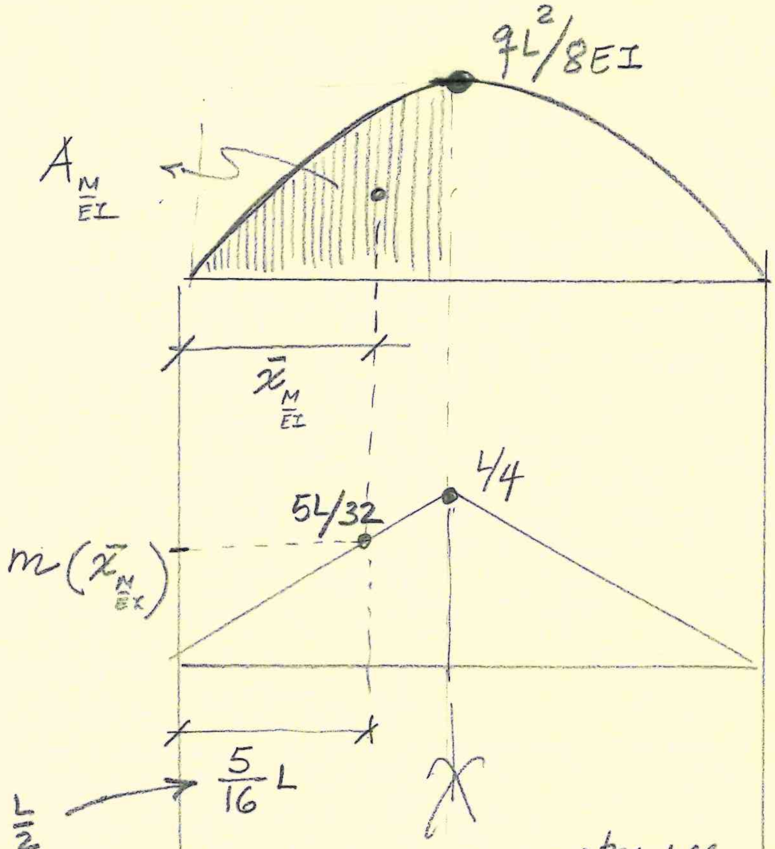
$$\delta = \frac{\cancel{10}^{\,5}}{\cancel{768}_{\,384}}\cdot\frac{qL^4}{EI} = \frac{5qL^4}{384EI} = \delta \quad (6)$$

$\rightarrow$ same as (5) from page 3.

METHOD #3 EVALUATE $\int \frac{M\bar{m}}{EI}dx$ as $A_{\left(\frac{M}{EI}\right)} \cdot \bar{m}\left(\bar{x}_{\frac{M}{EI}}\right)$

$A_{\frac{M}{EI}}$ = area under $\frac{M}{EI}$

$\bar{x}_{\frac{M}{EI}}$ = location of centroid of $\frac{M}{EI}$



NOTE: DUE TO SYMMETRY WE JUST CALCULATE ONE HALF AND MULT. BY 2

$$A = \frac{2}{3}\cdot\frac{qL^2}{8}\cdot\frac{L}{2}$$

$$A = \frac{qL^3}{24EI}$$

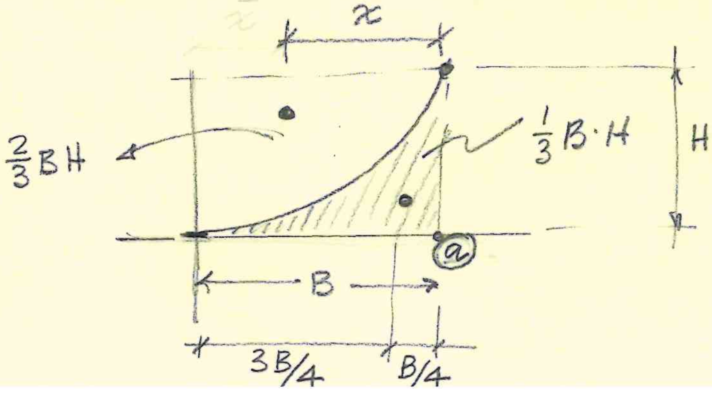
$$m(\bar{x}) = \frac{5}{8}\cdot\frac{L}{4} = \frac{5L}{32}$$

$$\delta = 2\cdot A\cdot m(\bar{x})$$

$$\boxed{\delta = \frac{5qL^4}{384EI}}$$

same as 5 and 6

NOTE: Remember for parabola



Take moment of area w.r.t. a

$$\cancel{B\cdot H}\cdot\frac{B}{2} = \frac{1}{3}\cancel{BH}\cdot\frac{B}{4} + \frac{2}{3}\cancel{BH}\cdot\bar{x}$$

$$\bar{x} = \left(\frac{1}{2}-\frac{1}{12}\right)\frac{3}{2}B = \frac{5}{12}\cdot\frac{3}{2}B$$

$$\boxed{\bar{x} = \frac{5B}{8}}$$