3.5) $k = m\omega_n^2 = m4\pi^2 f_n^2$

$f_n = 400$ cyc./min

$f_n = 400$ cyc/60 sec.

$f_n = 6.67$ cyc/s

$$k = \left(\frac{50\,lb}{386.4}\right) 4\pi^2 \left(6.67\ 1/s\right)^2$$

$k = 227.27$ lb/in

3.6) $T_0 = 0.35\,s$ $\qquad T_0 = 2\pi\sqrt{\frac{m}{k}}$ $\Rightarrow \boxed{\frac{T_0^2}{4\pi^2} \cdot k = m}$ (1)

$0.0031\,k = m$

Increase T_0 by 10%

(2) $\boxed{\frac{(1.10\,T_0)^2}{4\pi^2} k = m + \frac{2\,lb}{386.4\,\frac{in}{s^2}}}$

$0.0038 \cdot k = m + 0.0052\ \frac{s^2\text{-}lb}{in}$

sub (1) into (2)

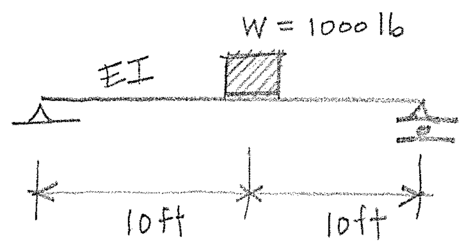
$(\cancel{s^2})\,0.0038\,k = (\cancel{s^2})\,0.0031\,k + 0.0052\ \frac{\cancel{s^2}\,lb}{in}$

$$\boxed{k = 7.43\ \frac{lb}{in}}$$

$m = (0.0031\,s^2)\ 7.43\ \frac{lb}{in}$

$$\boxed{m = 0.023\ lb\text{-}s^2/in}$$

3.11)



$$flex. = \frac{L^3}{48EI} = \frac{(20 \cdot 12)^3}{48\,(29{,}000\ ksi)(1830\ in^4)} = 0.0054\ in/kip$$

$$k = 1/flex = 184.27\ kip/in$$

$$m = \frac{W}{g} = \frac{1000\ lb}{386.4\ in/s^2} = \frac{1\ kip}{386.4\ in/s^2} = 0.0026\ kip\text{-}s^2/in$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{184.27}{0.0026}} = 267\ rad/s$$

$$\boxed{f_n = 42.60\ Hz.}$$

Maximum displacement:

$$X_{max} = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2} = \sqrt{(1.5)^2 + \left(\frac{4.5}{267}\right)^2} = 1.5\ in.$$

Maximum Velocity:

$$V_{max} = \omega_n X_{max} = 267 \cdot (1.5\ in) = 400.5\ in/s$$

Maximum Acceleration

$$A_{max} = \omega_n^2 X_{max} = (267)^2 (1.5\ in) = 276.74\ g's$$

3.12)

$W = 750 \text{ lb}$

$m = \frac{750}{386.4} \text{ lb-s}^2/\text{in} = 1.94 \text{ lb-s}^2/\text{in}$

$k = \frac{3EI}{L^3} = \frac{3(30{,}000{,}000 \text{ lb/in}^2)(2500 \text{ in}^4)}{[10(12)]^3}$

$k = 130{,}208.33 \text{ lb/in}$

a) $\omega_n = \sqrt{\frac{130{,}208}{1.94}} = 259 \text{ rad/s}$

$f_n = 41.23 \text{ Hz}.$

b) $x_0 = 2 \text{ in} \quad v_0 = 3 \text{ in/s}$

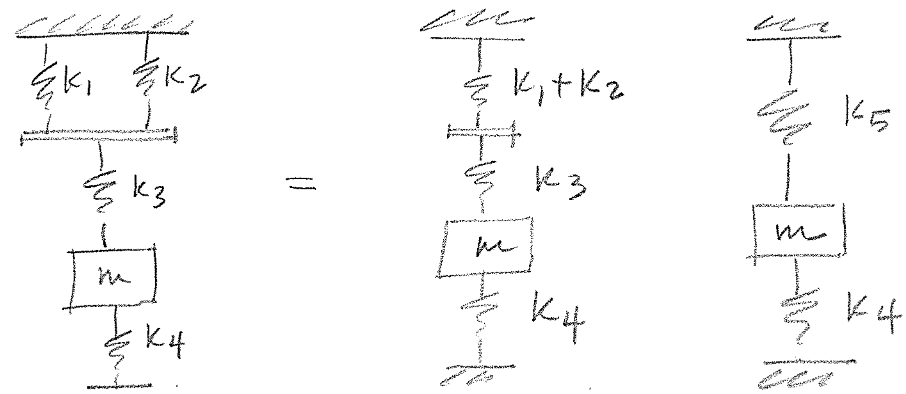
$X_{max} = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2} = \sqrt{2^2 + \left(\frac{3}{259}\right)^2} = 2 \text{ in}$

$V_{max} = \omega_n X_{max} = 259(2) = 518 \text{ in/s}$

$A_{max} = \omega_n^2 X_{max} = 134{,}164 \text{ in/s}^2 = 347 \text{ g's} \; !$

3.15)

(a)

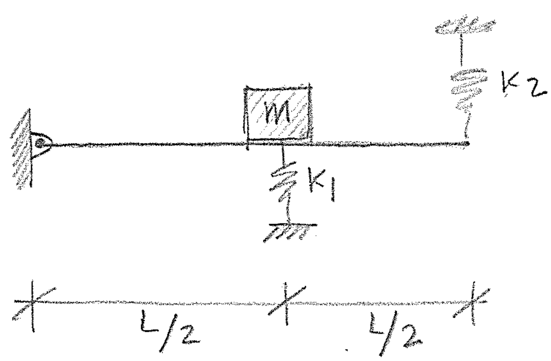


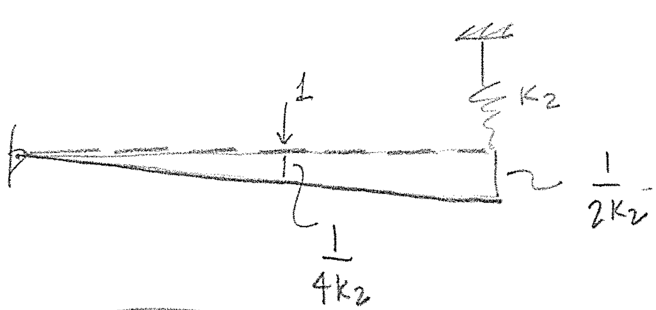
$$flex_5 = \frac{1}{k_1+k_2} + \frac{1}{k_3} = \frac{k_1+k_2+k_3}{(k_1+k_2)k_3}$$

$$k_5 = \frac{k_3(k_1+k_2)}{k_1+k_2+k_3}$$

$$k_e = k_4 + k_5 = \boxed{k_4 + \frac{k_3(k_1+k_2)}{k_1+k_2+k_3} = k_e}$$

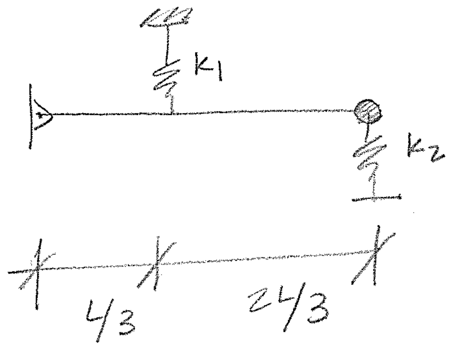
b)

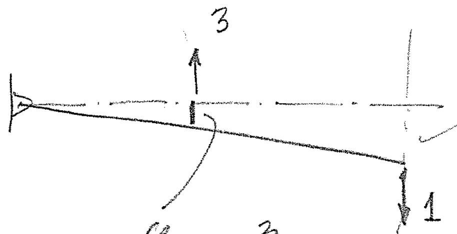




$$\boxed{k = k_1 + 4k_2}$$

c)



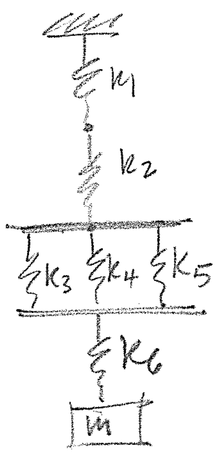


$\frac{9}{k_1} = flex.$

$flex = \frac{3}{k_1}$

$$\boxed{K_e = k_2 + \frac{k_1}{9}}$$

d)



$$\left.\begin{array}{l} \left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{1}{k_e} = \frac{k_1 + k_2}{k_1 k_2} \\ \\ \left(k_3 + k_4 + k_5\right) = k_e \end{array}\right\} K_0$$

$$\frac{1}{K_0} = \frac{K_1 + K_2}{K_1 K_2} + \frac{1}{k_3 + k_4 + k_5}$$

$$\frac{1}{K_0} + \frac{1}{K_6} = \frac{1}{K_{sys}} \Rightarrow \boxed{K_{sys} = \frac{K_0 K_6}{K_0 + K_6}}$$

3.19)

a) $I = I_o$

J_i = torsional inertia of circular shaft with diameter D_i

$$1/k_R = \frac{a}{GJ_1} + \frac{b}{GJ_2}$$

$$K_R = \frac{GJ_1J_2}{aJ_2 + bJ_1}$$

$$\omega_n = \sqrt{\frac{K_R}{I}} = \sqrt{\frac{GJ_1J_2}{I_o(aJ_2+bJ_1)}}$$

b) $$K_R = \frac{GJ_1}{L} + \frac{4GJ_2}{3L} = \frac{3GJ_1 + 4GJ_2}{3L}$$

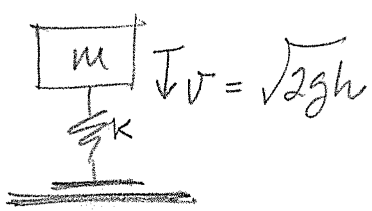
$$\omega_n = \sqrt{\frac{3GJ_1 + 4GJ_2}{3I_oL}}$$

c) $$K_R = \frac{GJ}{L} + KR^2$$

$$\omega_n = \sqrt{\frac{GJ + KR^2L}{I_oL}}$$

3.21) We already solved this problem in class.

At impact



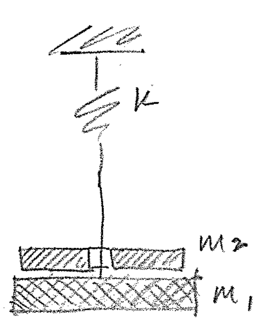
initial displ. $= x_0 = \frac{mg}{2K} (\uparrow)$

initial vel $= v_0 = \sqrt{2gh} (\downarrow)$

@ equilibrium point.

* note: if you want total displacement from the point of impact, then you need to add x_0 to $x(t)$

3.23)



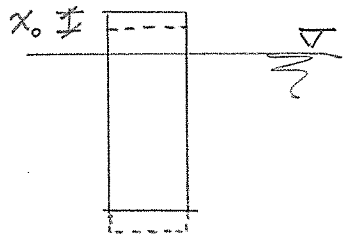
with respect to equilibrium point:

$$x_0 = \frac{m_2 g}{K} (\uparrow)$$

$$v_0 = \sqrt{2gh}\left(\frac{m_2}{m_1+m_2}\right) (\downarrow)$$

* (see note on previous problem).

3.31)



k = weight of displaced fluid per unit displacement (Archimedes!)

$$A \cdot 1 \cdot \rho g = k = A\rho g$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{A\rho_w g}{V \cdot \rho_c}} = \sqrt{H \cdot g\left(\frac{\rho_w}{\rho_c}\right)}$$

H = total height of cylinder

ρ_w = density of water

ρ_c = density of cylinder