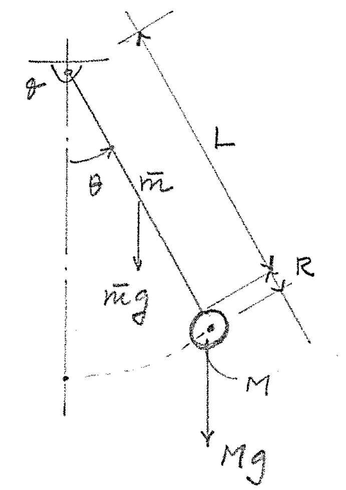
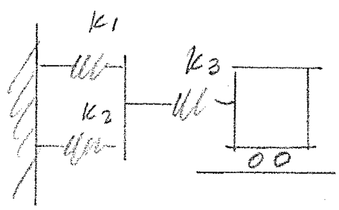
1.6)



$$\Sigma M^{(o)} = \bar{m} g L\left(\frac{L}{2}\right)\sin\theta + Mg\cdot(L+R)\sin\theta + \frac{1}{3}\bar{m}L^3\ddot{\theta} + (L+R)^2 M\ddot{\theta} = 0$$

$$0 = \left[\frac{1}{3}\bar{m}L^3 + (L+R)^2 M\right]\ddot{\theta} + \left[\frac{\bar{m}L^2}{2} + M(L+R)\right] g\sin\theta$$

1.12)

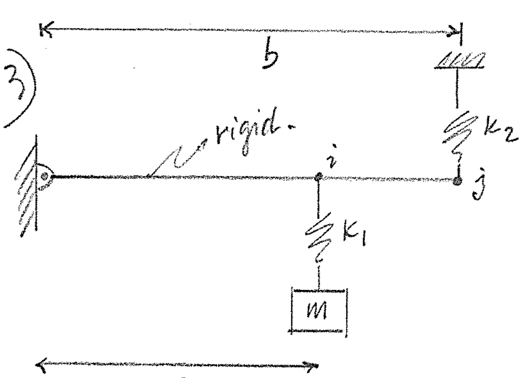


$$\frac{1}{K_e} = \frac{1}{k_1+k_2} + \frac{1}{k_3} = \frac{k_3}{k_3(k_1+k_2)} + \frac{(k_1+k_2)}{k_3(k_1+k_2)}$$

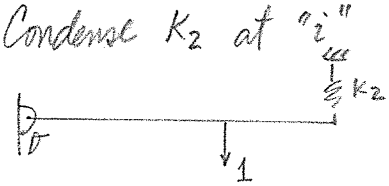
$$= \frac{k_1+k_2+k_3}{k_3(k_1+k_2)}$$

$$K_e = \frac{k_3(k_1+k_2)}{k_1+k_2+k_3}$$

1.13)



Condense K_2 at "i"



$$\Sigma M^{o} = 1\cdot a - F_2\cdot(b) = 0$$

$$F_2 = \frac{a}{b} \qquad \Delta_2 = \frac{a}{bK_2}$$

$$\frac{\Delta_1}{a} = \frac{\Delta_2}{b} \Rightarrow \Delta_1 = \frac{a^2}{b^2 K_2}$$

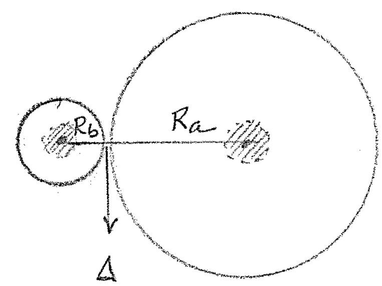
$$\frac{1}{\Delta_1} = K_{2e} = \left(\frac{b}{a}\right)^2 K_2$$

K_{2e} is in series with k_1.

$$K_e = \frac{1}{\frac{1}{K_1} + \frac{1}{K_{2e}}} = \frac{1}{\frac{1}{K_1} + \frac{a^2}{b^2}\cdot\frac{1}{K_2}} = \frac{1}{\frac{K_2 b^2 + K_1 a^2}{K_1 K_2 b^2}}$$

$$K_e = \frac{K_1 K_2 b^2}{K_2 b^2 + K_1 a^2}$$

1.14)



Compatibility:

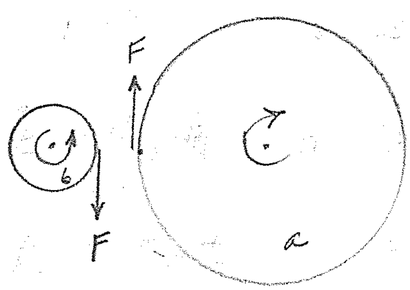
$$\theta_a \cdot R_a = \Delta = \theta_b R_b$$

$$\theta_b = \frac{R_a}{R_b}\,\theta_a = n\,\theta_a$$

(they rotate in opp. directions)

Rotational stiffness of shaft.

$$K = \frac{GJ}{L} \quad \text{(same for both)}$$



$$F \cdot R_a = K \cdot \theta_a$$

$$F = \frac{K \cdot \theta_a}{R_a}$$

Apply torque (T) at "b" then

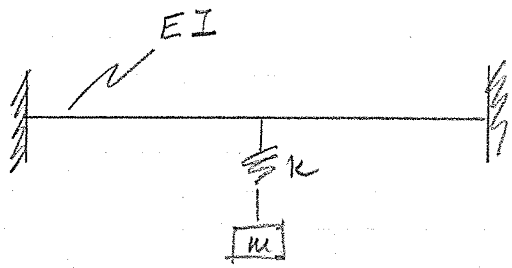
$$T = K \cdot \theta_b + F \cdot R_b$$

$$T = K\,\theta_b + K \cdot \theta_a \underbrace{\frac{R_b}{R_a}}_{1/n} = K\,\theta_b + K \cdot \frac{1}{n}\cdot\overbrace{\left(\frac{1}{n}\,\theta_b\right)}^{\theta_a} = K\theta_b\left(1 + \frac{1}{n^2}\right)$$

$$T = \underbrace{K\left(1+\frac{1}{n^2}\right)}_{\text{equivalent stiffness, at "b".}} \theta_b$$

Same analysis can be carried out if torque is applied at "a".

1.29)



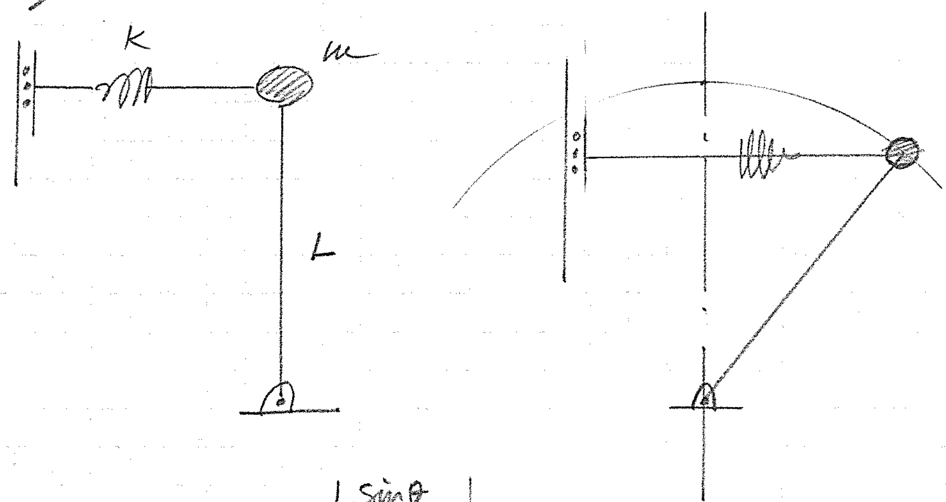
The beam and the spring are in series (because def. adds)

$$K_{beam} = \frac{192EI}{L^3}$$

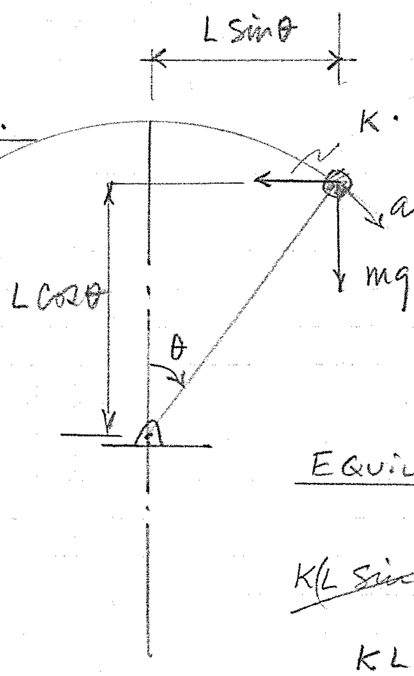
$$K_{sys} = \frac{1}{\frac{L^3}{192EI} + \frac{1}{k}} = \frac{1}{\frac{kL^3}{192EIk} + \frac{192EI}{k\,192EI}} = \frac{192EI \cdot k}{192EI + kL^3}$$

$\boxed{m\ddot{x} + K_{sys}x = 0}$ → Eq. of motion.

1.32)



FBD.



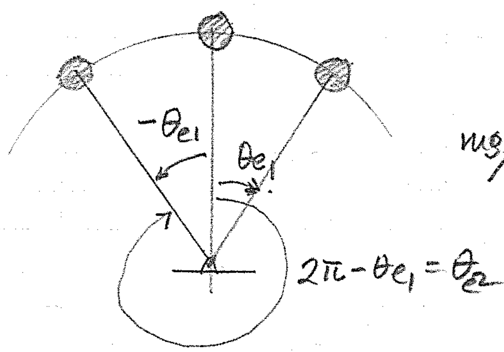
EQ. OF MOTION

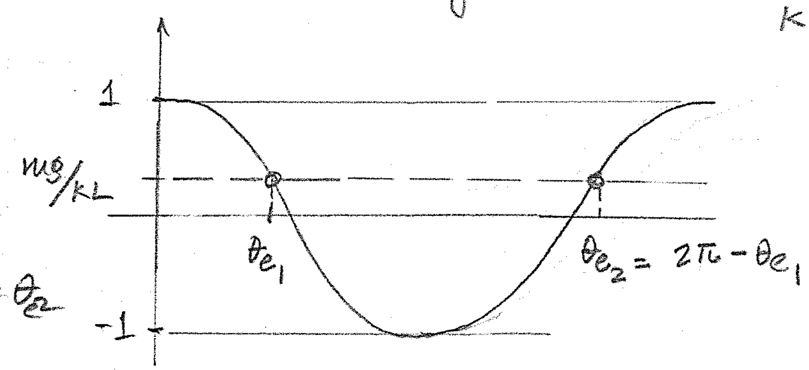
$$\Sigma M^{(0)} = (K \cdot L \sin\theta)(L \cdot \cos\theta) - mg \cdot L\sin\theta + mL^2\ddot{\theta} = 0$$

EQUIL. POINTS. $\dot{\theta} = \ddot{\theta} = 0$

$$K\cancel{(L\sin\theta)}(L\cos\theta) - mg\cancel{(L\sin\theta)} = 0$$

$$KL\cos\theta = mg \Rightarrow \cos\theta = \frac{mg}{KL}$$





$$\theta_{e_1} = \cos^{-1}\left(\frac{mg}{KL}\right)$$

if $mg > KL$ no equilibrium points except for $\theta = 0$.

if $mg < KL$ there are 3 equil. points
$\theta = 0$, $\theta = \theta_{e_1}$
$\theta = \theta_{e_2}$

EQUATION OF MOTION @ $\theta = 0$

$$mL^2\ddot{\theta} + \left(KL^2 - mgL\right)\theta = 0$$

EQUATION OF MOTION @ $\theta_{e_1} = \cos^{-1}\left(\frac{mg}{KL}\right)$

$$mL^2\ddot{\theta} + KL^2 \underbrace{\frac{d}{d\theta}\left[\sin\theta \cdot \cos\theta\right]_{\theta=\theta_{e_1}}}_{\text{tangent @ } \theta=\theta_{e_1}} \cdot \theta - mgL \underbrace{\frac{d}{d\theta}\left[\sin\theta\right]_{\theta=\theta_{e_1}}}_{\text{tangent @ } \theta=\theta_{e_1}} \theta = 0$$