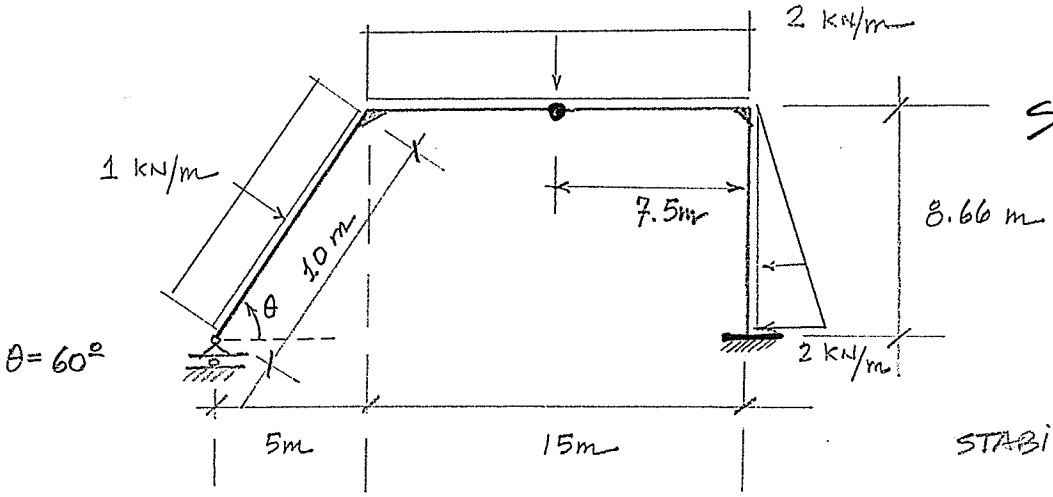


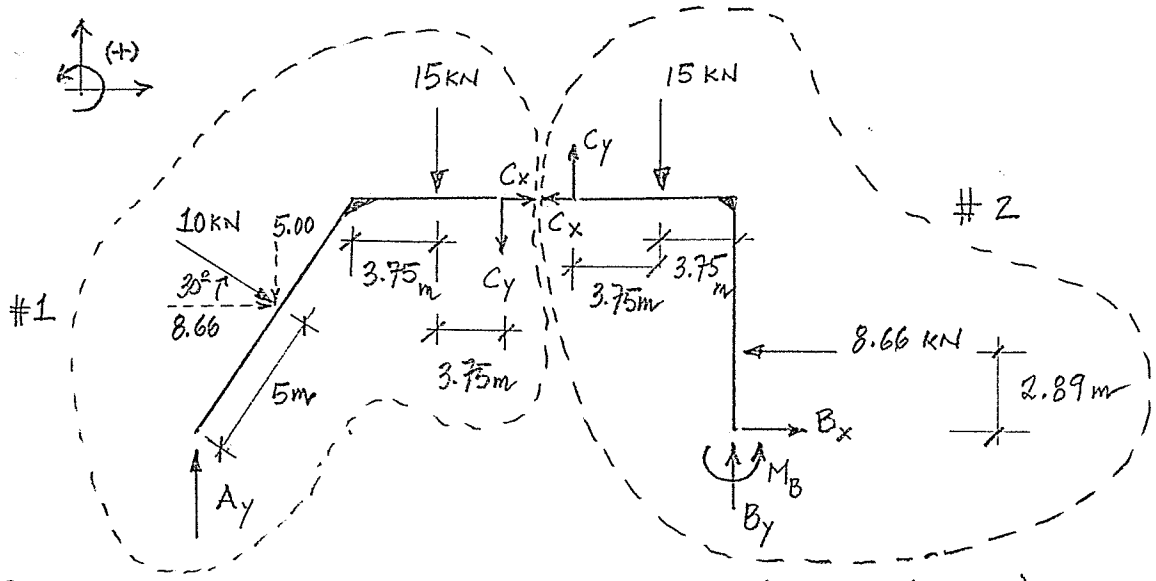
stable!

STABILITY CHECK

$3e = 6$
$s = 4$
$c = 2$
0

STATICALLY DET.

FBD to compute reactions.



#1

$\Sigma M_c = 15(3.75) + 5(7.5+2.5) + 8.66(4.33) - A_y(7.5+5) = 0$

$\boxed{A_y = 11.50 \text{ kN}}$

$\Sigma F_y = A_y - 5 - 15 - C_y = 0 \Rightarrow C_y = 11.50 - 5 - 15 \Rightarrow \boxed{C_y = -8.50 \text{ kN}}$

$\Sigma F_x = 8.66 + C_x = 0 \Rightarrow \boxed{C_x = -8.66 \text{ kN}}$

#2

$\Sigma M_B = 15(3.75) + 8.50(7.50) + 8.66(2.89) - 8.66(8.66) + M_B = 0$

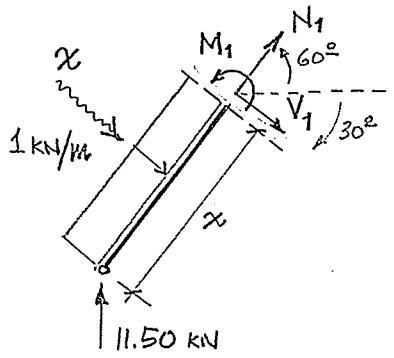
$\boxed{M_B = -70.03 \text{ kN}}$

$\Sigma F_y = -8.50 - 15 + B_y = 0 \Rightarrow \boxed{B_y = 23.5 \text{ kN}}$

$\Sigma F_x = 8.66 - 8.66 + B_x = 0$

$\boxed{B_x = 0}$

INTERNAL FORCES:



$\Sigma F_x = x\cos 30 + N_1 \cos 60 + V_1 \cos 30 = 0$

$\Sigma F_y = x\sin 30 + N_1 \sin 60 - V_1 \sin 30 + 11.50 = 0$

multiply (ΣF_x) by $\sin 30$ and (ΣF_y) by $\cos 30$ to eliminate V_1

$$\sin 30\left[x\cos 30 + N_1 \cos 60 + V_1 \cos 30\right] + \cos 30\left[x\sin 30 + N_1 \sin 60 - V_1 \sin 30 + 11.50\right] = 0$$

$$N_1\left[\sin 30 \cos 60 + \cos 30 \sin 60\right] = -11.50\,(\cos 30)$$

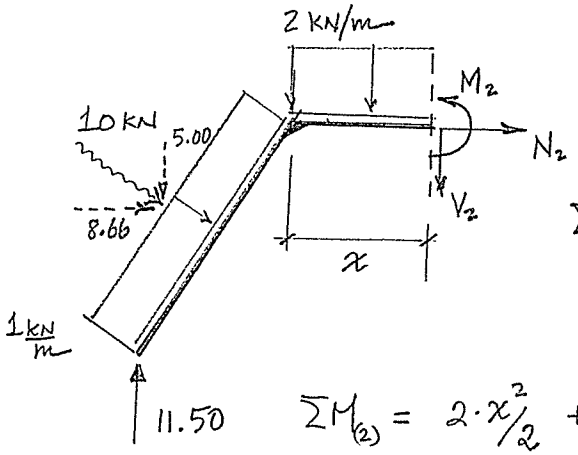
$$N_1 = \frac{-11.50\,(0.86)}{(0.5)(0.5) + (0.86)(0.86)} = \boxed{-9.89 \text{ kN} = N_1}$$

From ΣF_x:

$$x\cos 30 + (-9.89)\cos 60 + V_1 \cos 30 = 0$$

$$\boxed{V_1 = -x + 5.75}$$

$$\Sigma M_{(1)} = \frac{x^2}{2} + M_1 - 11.50\,(x\cos 60) = 0 \Rightarrow \boxed{M_1 = 5.75x - \frac{x^2}{2}}$$



$\Sigma F_x = 8.66 + N_2 = 0 \Rightarrow \boxed{N_2 = -8.66}$

$\Sigma F_y = 11.50 - 5.00 - 2 \cdot x - V_2 = 0$

$\boxed{V_2 = 6.5 - 2x}$

$\Sigma M_{(2)} = 2 \cdot \frac{x^2}{2} + 5\left(2.5 + x\right) + 8.66\left(\frac{8.66}{2}\right) - 11.50\left(5 + x\right) + M_2 = 0$

$\boxed{M_2 = -x^2 + 6.5x + 7.50}$

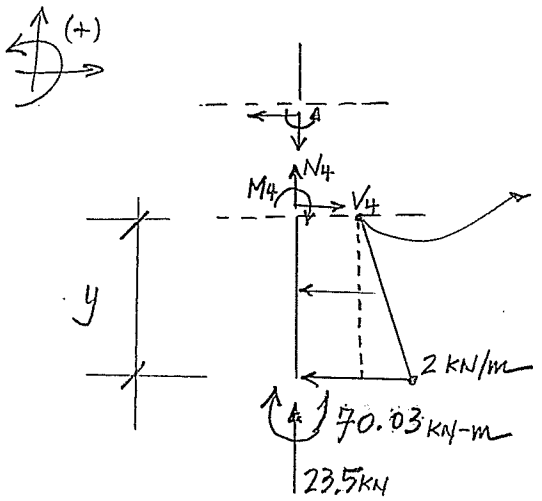
Verification:

@ $x = 7.5$ M_2 must be zero (since there is a hinge)

$M_2(x=7.5) = -(7.5)^2 + 6.5(7.5) + 7.5 = 0$ ok.

$V_2(x=7.5) = 6.5 - 2(7.5) = -8.5$ This must be equal to C_y

$N_2(x=7.5) = -8.66$ This must be equal to C_x



$$\frac{2}{8.66} = \frac{q(y)}{8.66-y} \Rightarrow q(y) = \frac{2(8.66-y)}{8.66}$$

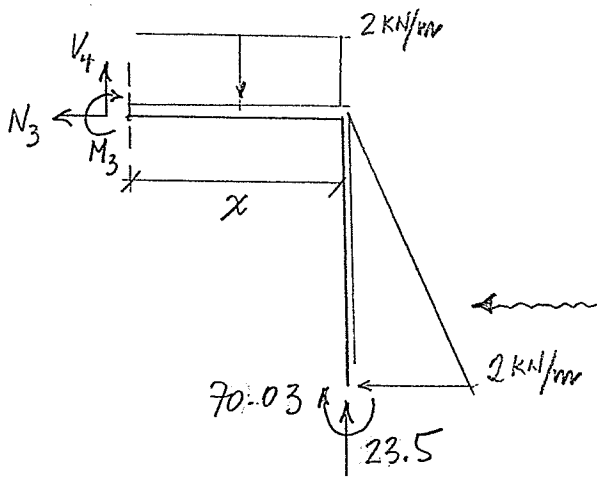
$$q(y) = 2 - 0.115y$$

$$\Sigma F_x = V_4 - [2 + (2 - 0.115y)] \cdot \frac{y}{2} = 0 \Rightarrow \boxed{V_4 = 2y - 0.0575y^2}$$

$$\Sigma F_y = N_4 + 23.5 = 0 \Rightarrow \boxed{N_4 = -23.5}$$

$$\Sigma M_{(4)} = -M_4 - 70.03 - (2 - 0.115y)\frac{y^2}{2} - (0.115y)\left(\frac{y}{2}\right)\cdot\frac{2}{3}y = 0$$

$$\boxed{M_4 = -70.03 - y^2 + 0.019y^3}$$



$$\Sigma F_x = -N_3 - 8.66 = 0$$

$$\boxed{N_3 = -8.66}$$

$$\Sigma F_y = V_4 - 2x + 23.5 = 0$$

$$\boxed{V_4 = 2x - 23.5}$$

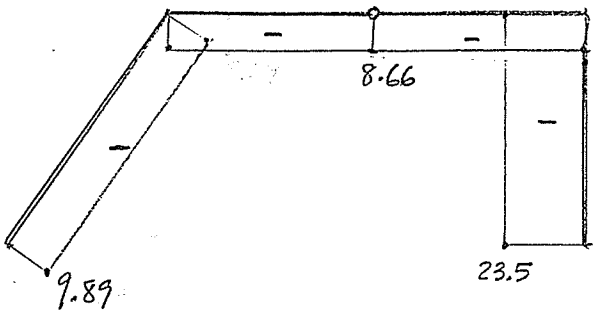
$$\Sigma M_{(3)} = -M_3 - 2x^2/2 - 8.66(5.77) - 70.03 + 23.5x = 0$$

$$\boxed{M_3 = -x^2 + 23.5x - 120}$$

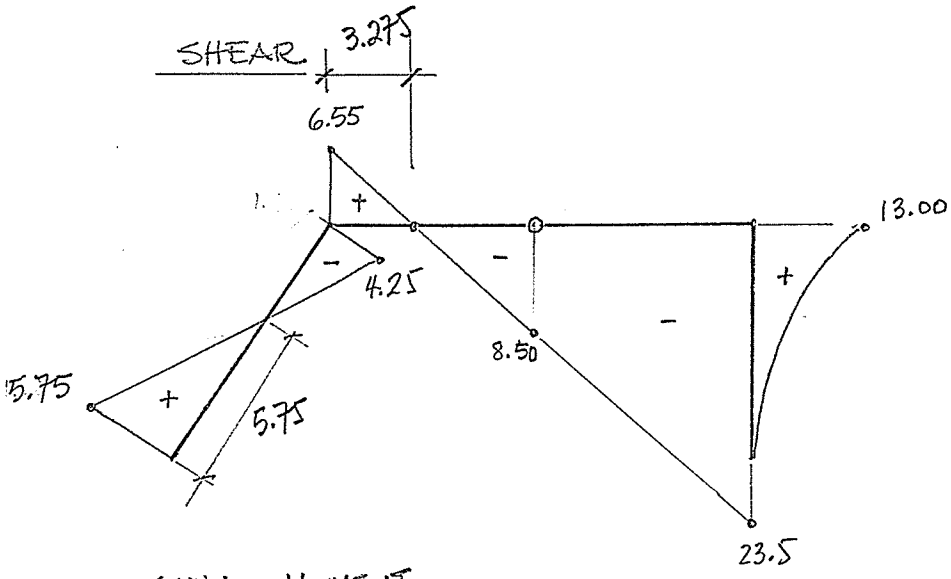
verification:

@ $x = 7.5$ $M_3 = 0$ ok ✓

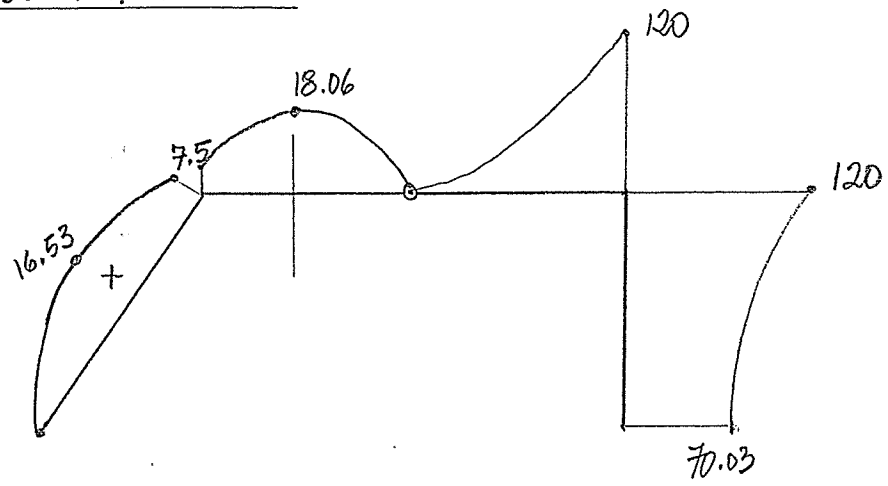
AXIAL

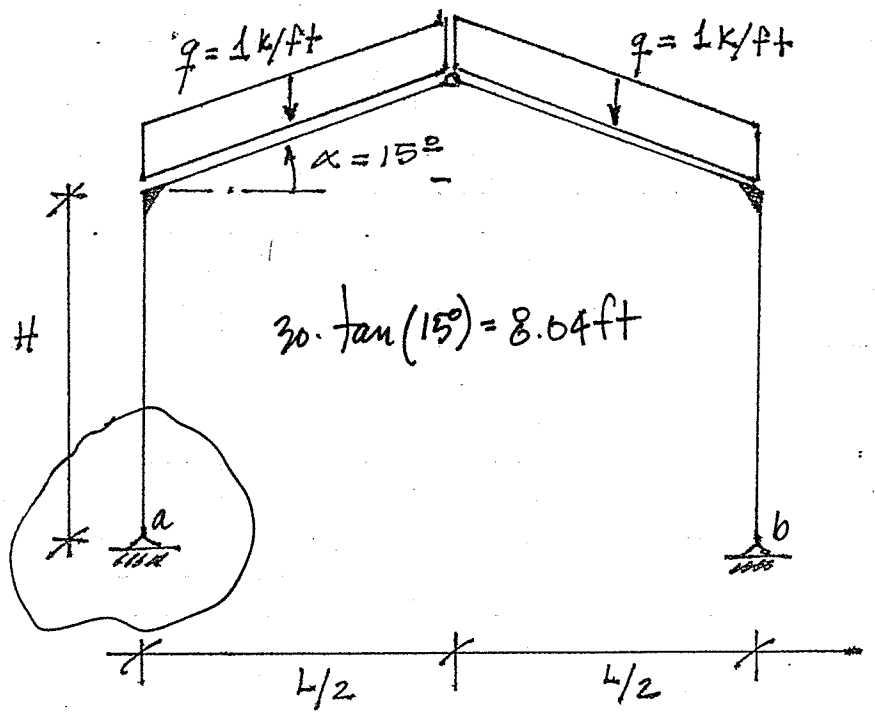


SHEAR



BENDING MOMENT

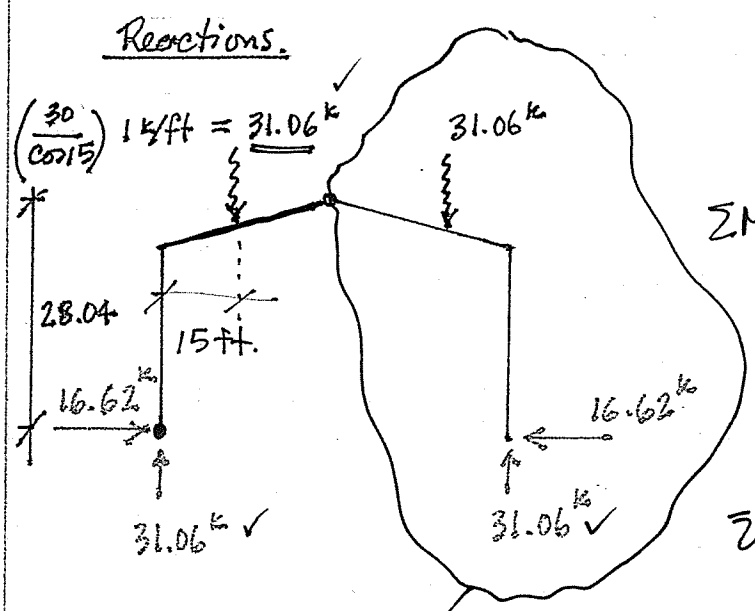




L = 60 ft.

H = 20 ft

Reactions.



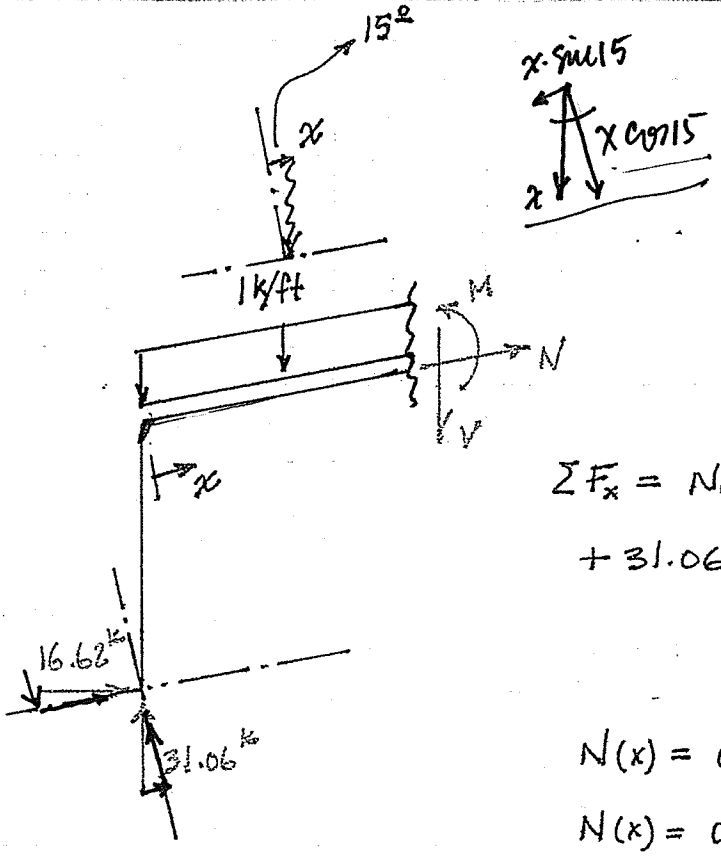
$\Sigma M^{(a)} = 31.06(15) + 31.06(45) - R_b(60) = 0$

$R_b = 31.06^k$

$\Sigma F =$

$\Sigma M^{hinge} = 31.06(15) - 31.06(30) + R_{bx}(28.04)$

$R_{bx} = 16.62^k$



$$\Sigma F_x = N(x) - x(\sin 15) + 16.62(\cos 15) + 31.06(\sin 15) = 0$$

$$N(x) = 0.26x - 16.05 - 8.04$$

$$N(x) = 0.26x - 24.09$$

$$\Sigma F_y = -V(x) - x(\cos 15) - 16.62(\sin 15) + 31.06(\cos 15) = 0$$

$$V(x) = -0.97x + 25.70$$

$$\Sigma M = -x\left(x \cdot \cos 15/2\right) + 31.06(x \cdot \cos 15) - 16.62(20 + x \sin 15) = M(x)$$

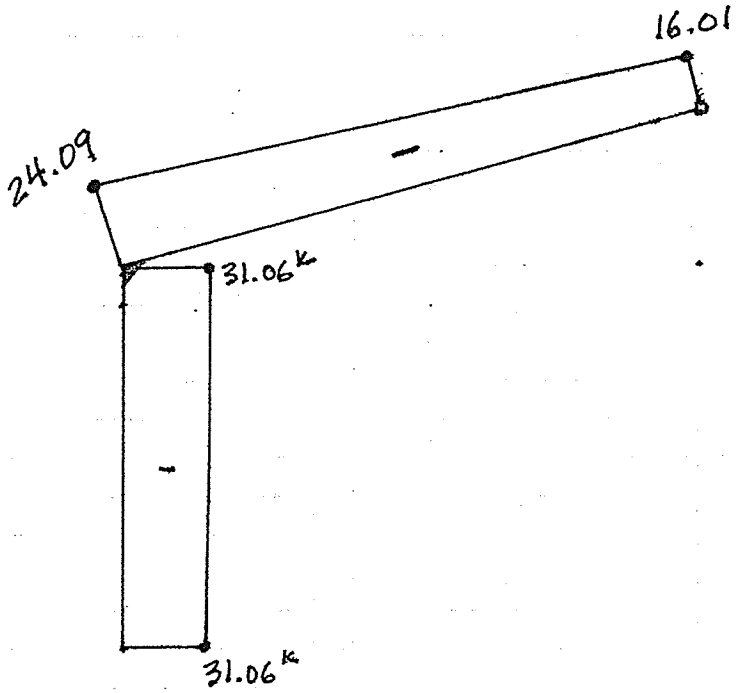
$$M(x) = -0.48x^2 + 30x - 332.4 - 4.30x$$

$$M(x) = -0.48x^2 + 25.7x - 332.4$$

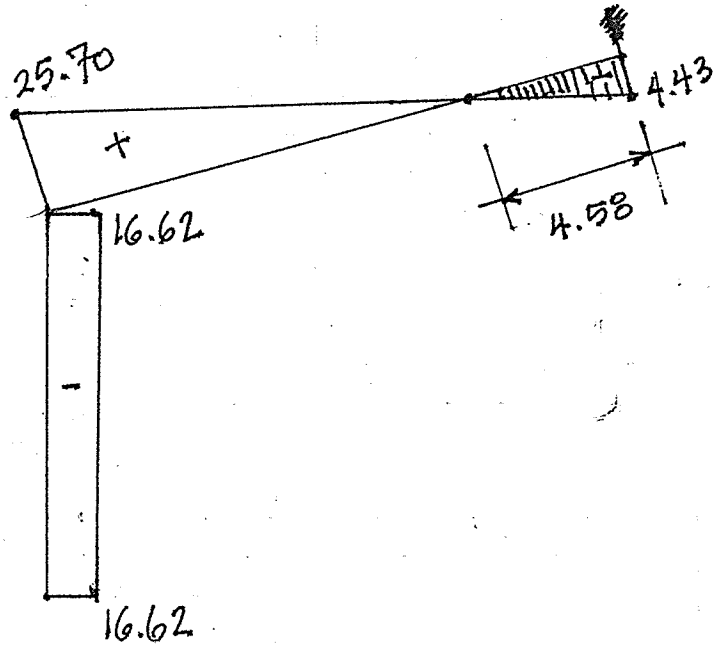
$$x_h = \frac{30}{\cos 15} = 31.06$$

$$M(x_h) \cong 0 \quad \checkmark$$

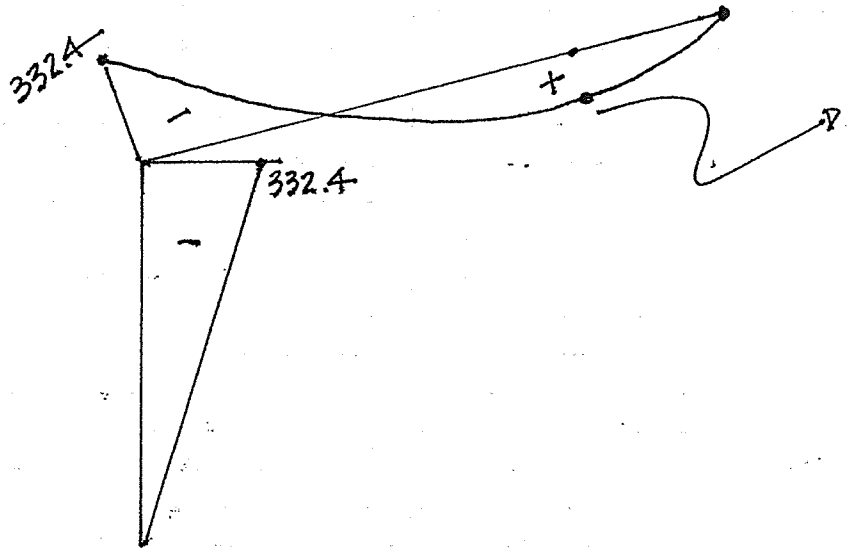
AXIAL : (k)



SHEAR : (k)

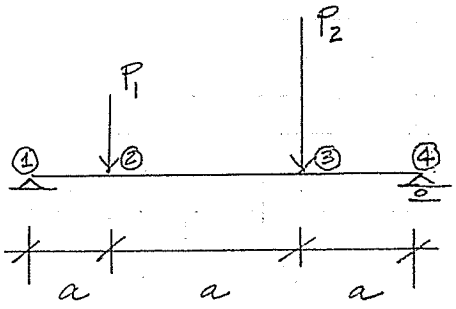


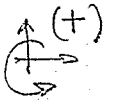
BENDING MOMENT: (k-ft)



$M^{+}_{MAX} = \frac{1}{2}(4.43)(4.58)$

$M^{+}_{MAX} = 10.1$ k-ft





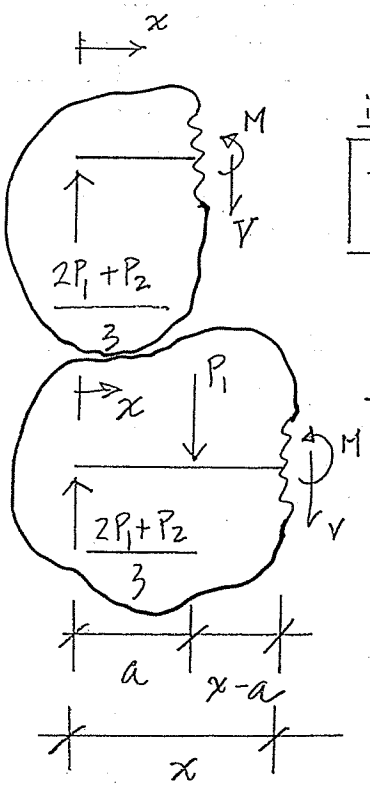
$$\sum M^{(4)} = -P_1(2a) - P_2(1a) + R_{4y}(3a) = 0$$

$$R_{4y} = \frac{P_1 + 2P_2}{3}$$

$$\sum M^{(1)} = 1aP_2 + 2aP_1 - R_{1y}(3a) = 0$$

$$R_{1y} = \frac{2P_1 + P_2}{3}$$

$$R_{1y} + R_{2y} - P_1 - P_2 = 0 \quad \checkmark$$



if $0 \le x < a$

$$\boxed{V(x) = \frac{2P_1 + P_2}{3}}$$

$$\boxed{M(x) = \left(\frac{2P_1 + P_2}{3}\right)x}$$

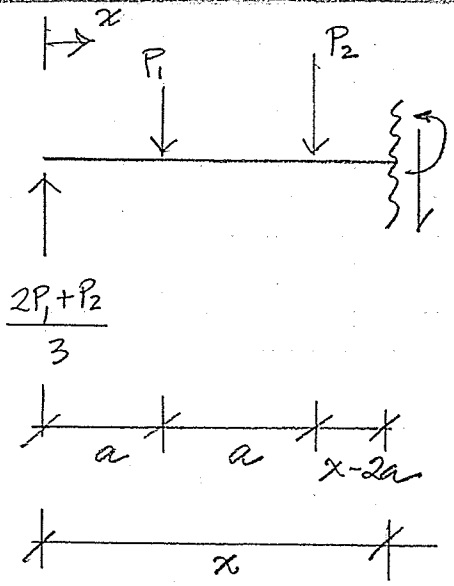
if $a \le x < 2a$

$$V(x) = \frac{2P_1}{3} + \frac{P_2}{3} - P_1 = \boxed{\frac{P_2 - P_1}{3} = V(x)}$$

$$M(x) = \frac{2P_1}{3}x + \frac{P_2}{3}x - P_1(x-a)$$

$$= \frac{2P_1}{3}x + \frac{P_2}{3}x - P_1x + P_1a$$

$$\boxed{M(x) = P_1a + \left(\frac{P_2 - P_1}{3}\right)x}$$



$$V(x) = \frac{2P_1}{3} + \frac{P_2}{3} - P_1 - P_2$$

$$\boxed{V(x) = -\left(\frac{P_1 + 2P_2}{3}\right)}$$

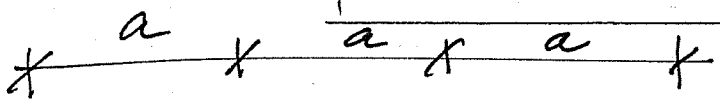
$$M(x) = \frac{2P_1}{3}x + \frac{P_2}{3}x - P_1(x-a)$$

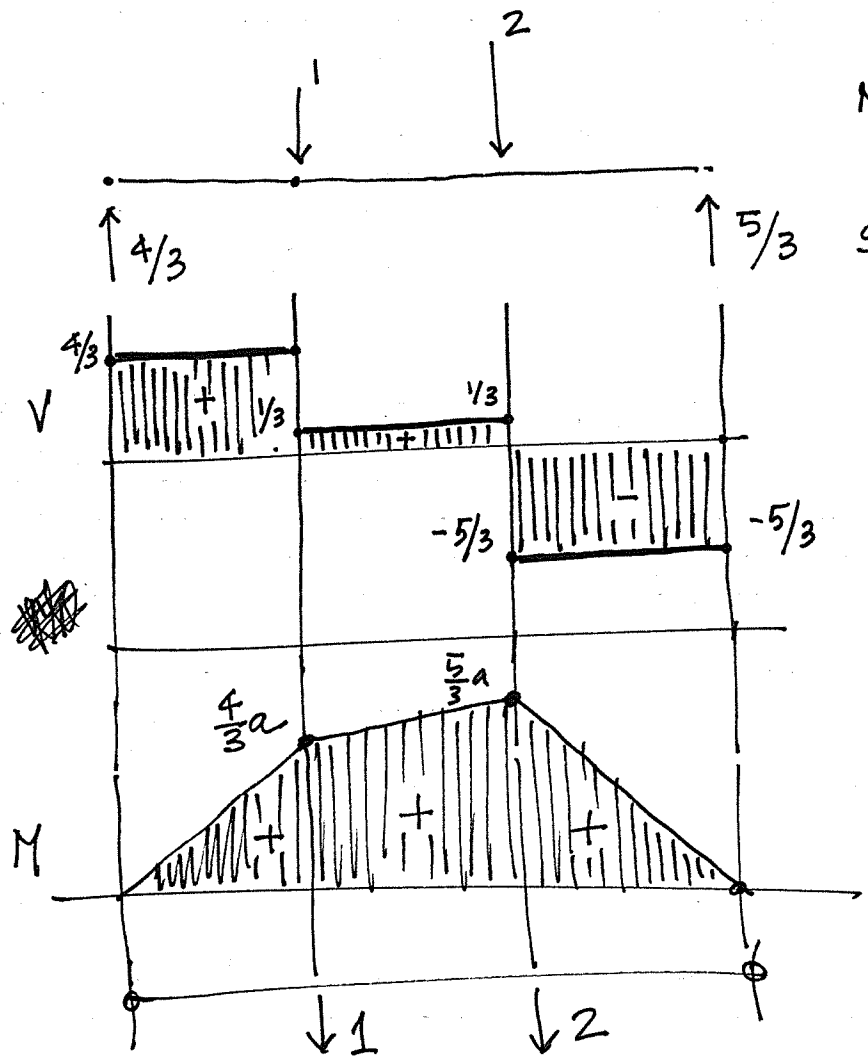
$$- P_2(x-2a)$$

$$= \frac{2P_1}{3}x + \frac{P_2}{3}x - P_1 x + P_1 a$$

$$- P_2 x + 2P_2 a$$

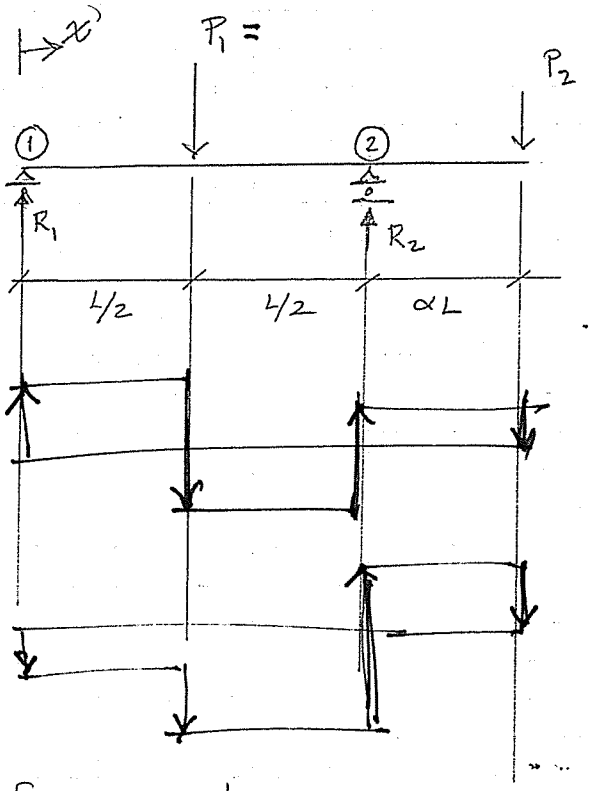
$$\boxed{M(x) = (P_1 + 2P_2)a - \left(\frac{P_1 + 2P_2}{3}\right)x}$$





$$M_x = \int_0^x V(\tau)\, d\tau$$

$$\frac{dM}{dx} = V$$



$$\Sigma M^{①} = P_1 \frac{L}{2} - R_2 L + P_2 (1+\alpha) L = 0$$

$$\boxed{R_2 = \frac{P_1}{2} + P_2 (1+\alpha)}$$

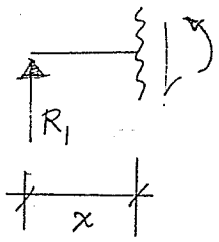
$$\Sigma F = R_1 - P_1 + R_2 - P_2 = 0$$

$$R_1 = P_1 + P_2 - R_2$$

$$R_1 = P_1 - \frac{P_1}{2} - P_2 (1+\alpha) - P_2$$

$$\boxed{R_1 = \frac{P_1}{2} - \alpha P_2}$$

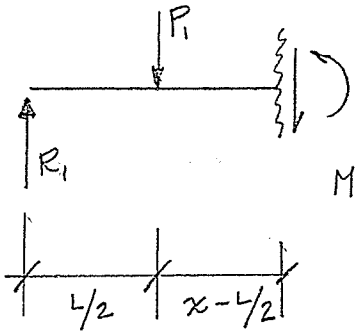
for $0 \le x < \frac{L}{2}$



$$V(x) = R_1$$

$$M(x) = R_1 \cdot x$$

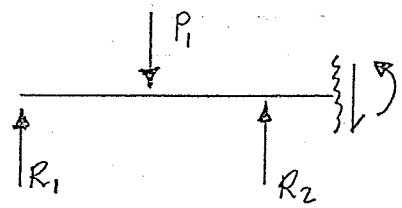
for $\frac{L}{2} \le x < L$



$$V(x) = R_1 - P_1$$

$$M(x) = R_1 x - P_1 \left(x - \frac{L}{2}\right)$$

for $L \le x < (1+\alpha) L$



$$V(x) = R_1 - P_1 + R_2$$

$$M(x) = R_1 x - P_1 \left(x - \frac{L}{2}\right) + R (x - L)$$

CE-170 Structural Analysis

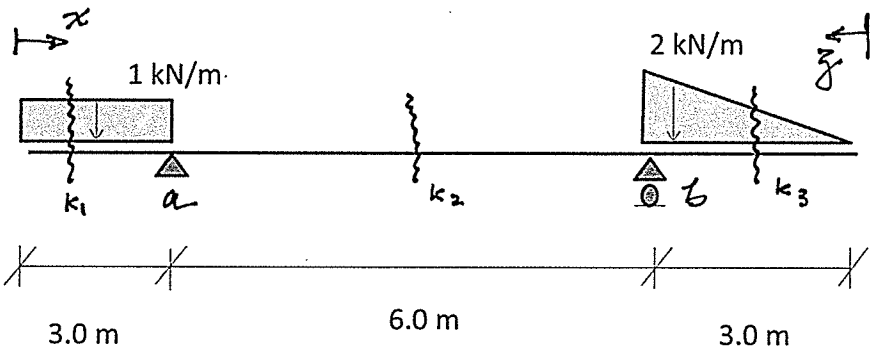
Instructor: Eric M. Hernandez, Ph.D.

NAME: SOLUTIONS

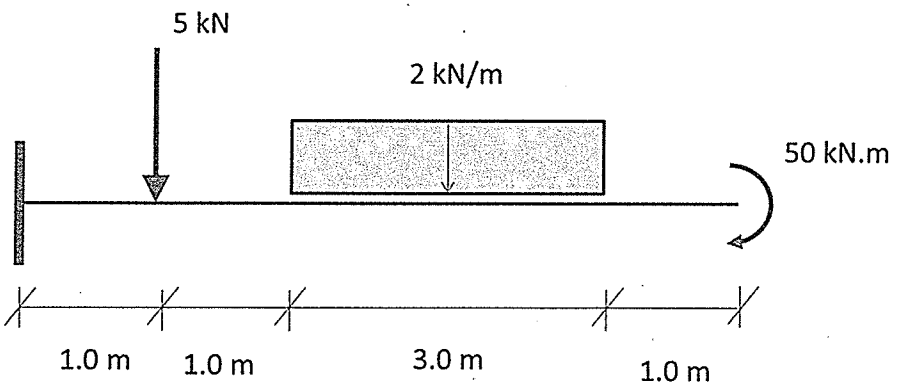
Exam #3

1. Determine the equations that describe the internal bending moment and shear force in the beam shown below. Determine the maximum bending moment and its location along the axis of the beam. (60%)

$z = 12 - x$

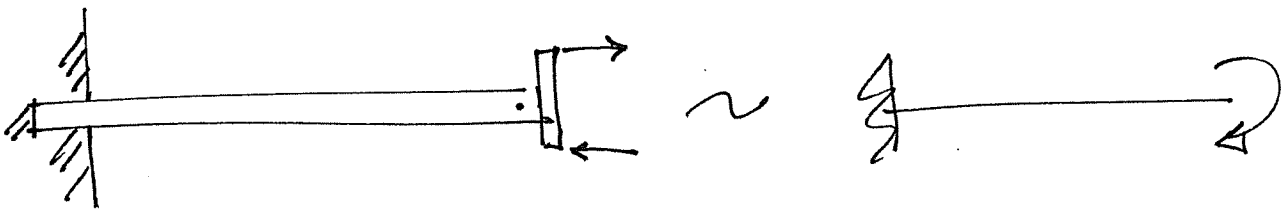


2. Draw the shear and moment diagram of the beam shown below. Show values of locations of interest. (40%)



Bonus Question (5 pts)

A car travels from Burlington to Boston at an average speed of 70 mph and then comes back at an average speed of 50 mph. What was the average speed for the whole trip?



SOLUTIONS - EXAM # 3
ERIC M. HERNANDEZ

1:)

Reactions

$\rightarrow \Sigma M^{(a)}$

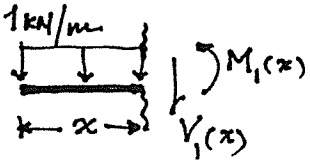
$$R_b = \left[\frac{2(3)}{2}\cdot(6+1) - 1(3)\cdot 1.50\right]\frac{1}{6.00} = \boxed{2.75\ kN = R_b}$$

$$R_a = 3(1) + 2(3)/2 - 2.75 = \boxed{3.25\ kN = R_a}$$

$\rightarrow \Sigma F_y$

SECTION CUTS:

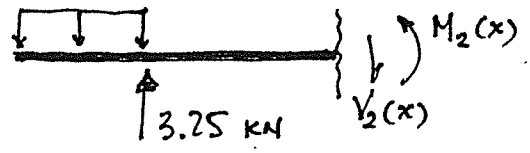
K_1 $\quad 0 \le x \le 3$



$$V_1(x) = -1\cdot x = \boxed{-x = V_1(x)}$$

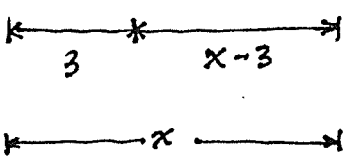
$$M_1(x) = -1\cdot x^2/2 = \boxed{\frac{-x^2}{2} = M_1(x)}$$

K_2 $\quad 3 \le x \le 6$



$$\Sigma F_y = -3 + 3.25 - V_2(x) = 0$$

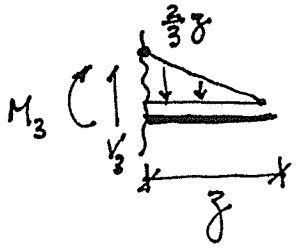
$$\boxed{V_2(x) = 0.25}$$



$$\Sigma M^{K_2} = -M_2(x) + 3.25(x-3) - 3(x-1.50) = 0$$

$$\boxed{M_2(x) = 0.25x - 5.25}$$

K_3 $\quad 0 \le z < 3$ $\qquad z = 12 - x$

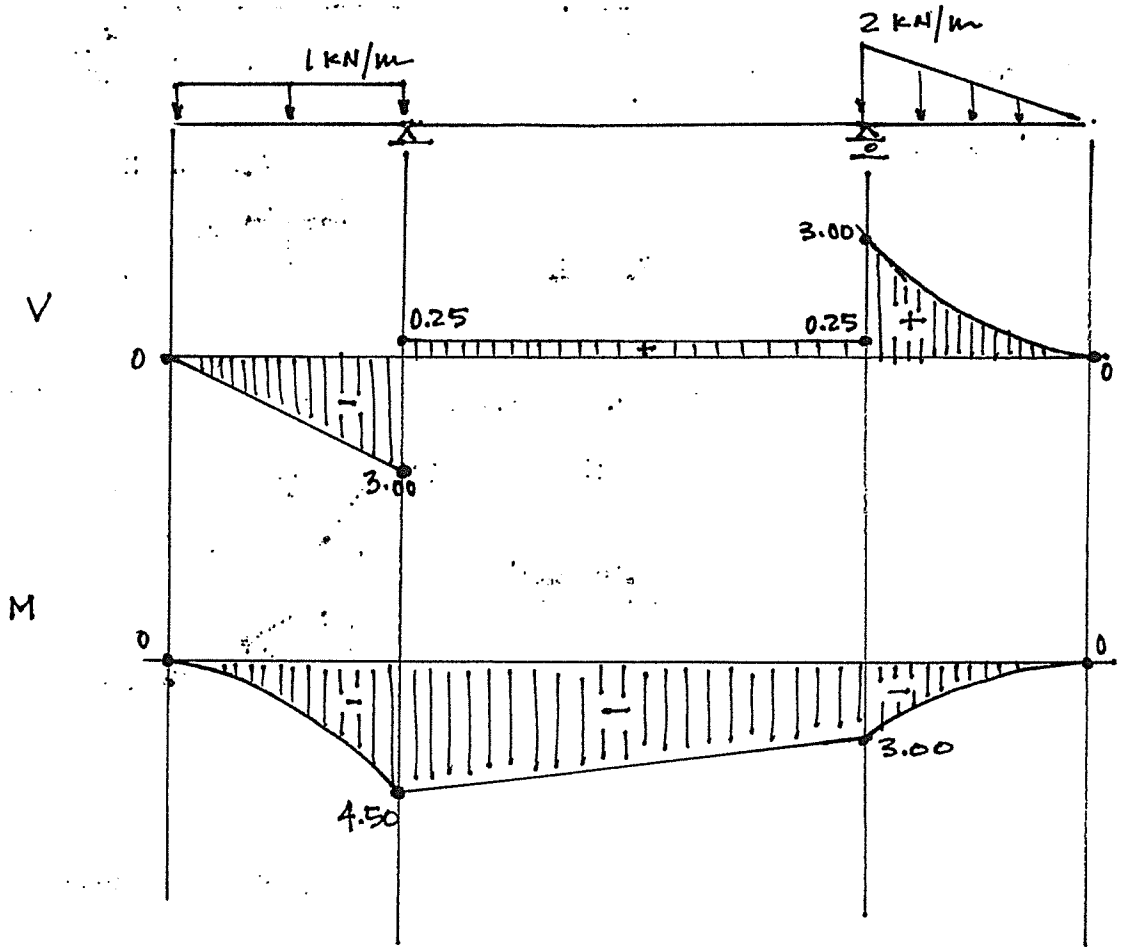


$$\Sigma F_y = -\left(\tfrac{2}{3}z\right)z/2 + V_3(z) = 0 \Rightarrow V_3(z) = z^2/3$$

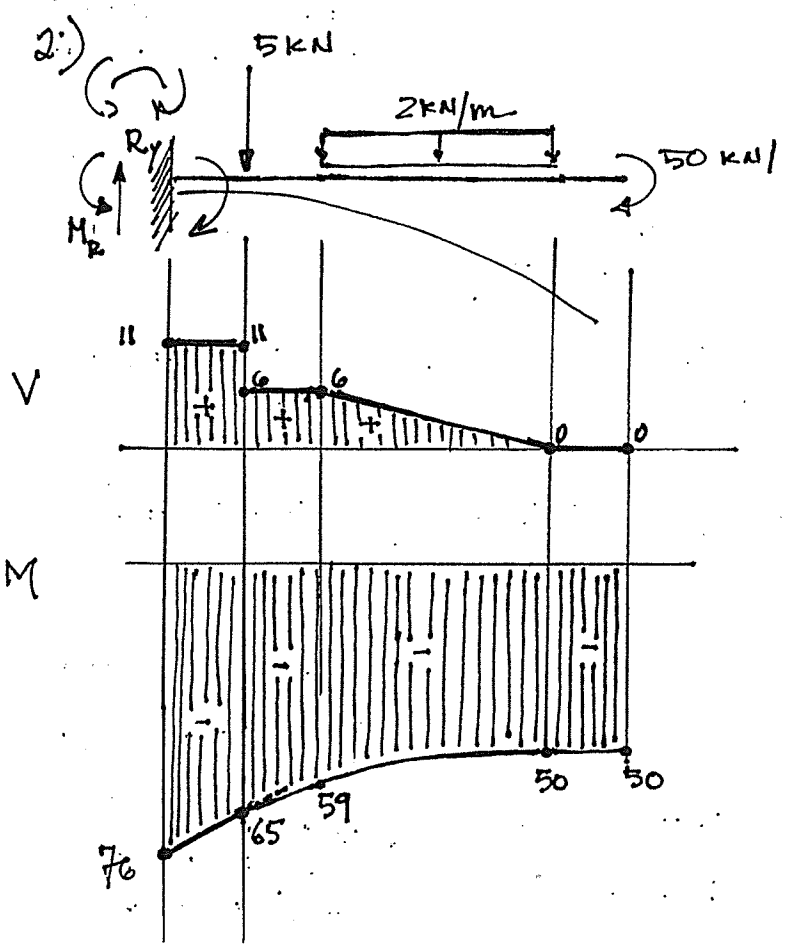
$$\boxed{V_3(x) = (12-x)^2/3}$$

$$\Sigma M_{K_3} = \left(z^2/3\right)\cdot\frac{z}{3} = z^3/9 \Rightarrow \boxed{\frac{(12-x)^3}{9} = M_3(x)}$$

Moment and Shear Diagrams.



2.)



$R_y = 5 + 2(3) = 11$ KN //

$M_R = 5(1) + 2(3)(3.5) + 50$

$M_R = 76$ KN·m //

Bonus Question:

Bur → Bos $\quad t_1 = \frac{x}{70}$

Bos → Bur $\quad t_2 = \frac{x}{50}$

$x \Rightarrow$ dist from Bur → Bos

$t_1 \Rightarrow$ travel time from Bur → Bos

$t_2 \Rightarrow$ " " " Bos → Bur

total time:

$$t_T = t_1 + t_2 = \frac{x}{70} + \frac{x}{50} = \frac{120x}{3500}$$

total dist:

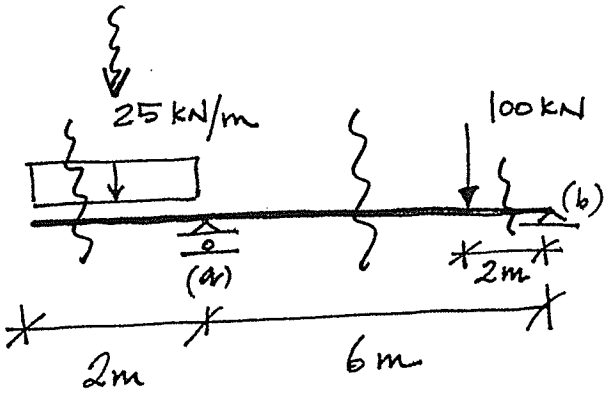
$x_T = 2x$

Average speed $= \frac{x_T}{t_T} = (2x)(3500)/120x =$ **58.33 mph**

MIDTERM EXAM

SOLUTIONS

1°) (50)



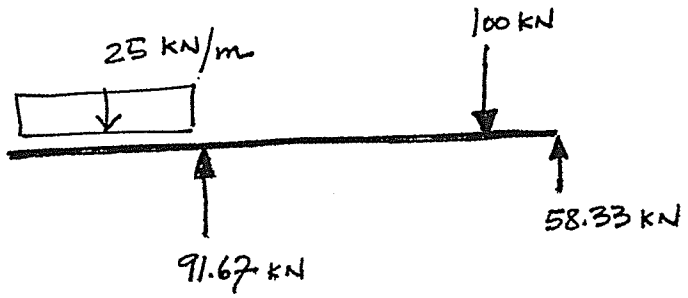
$$A_y = \frac{100(2) + 25(2)(6+1)}{6}$$

$$A_y = 91.67\ kN$$

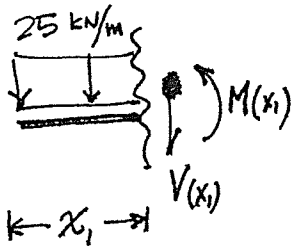
$$B_y = -A_y + 25(2) + 100$$

$$B_y = -91.67 + 50 + 100$$

$$B_y = 58.33\ kN$$



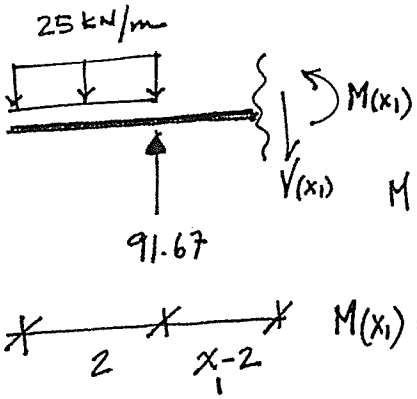
$0 \le x_1 < 2$



$$V(x_1) = -25x_1$$

$$M(x_1) = -12.5x_1^2$$

$2 \le x_1 < 6$



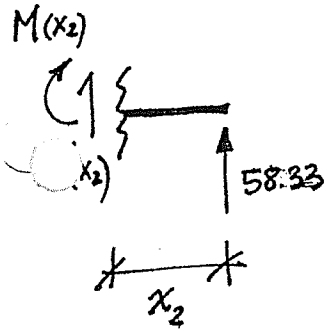
$$V(x_1) = 91.67 - 50$$

$$V(x_1) = 41.67$$

$$M(x_1) = 91.67(x_1 - 2) - 50(x_1 - 1)$$

$$M(x_1) = 41.67x_1 - 133.34$$

$0 \le x_2 < 2$

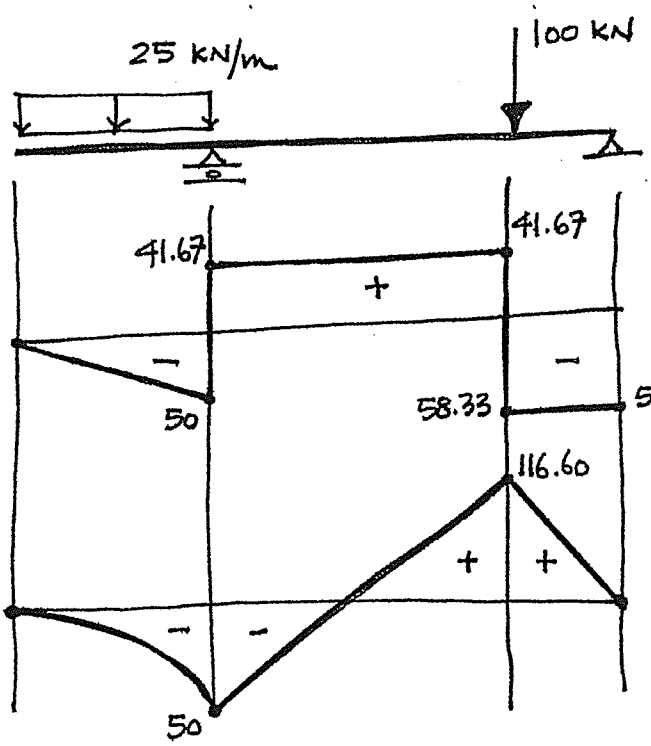


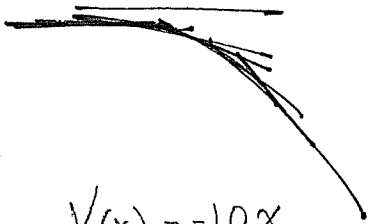
$$V(x_2) = -58.33$$

$$M(x_2) = +58.33x_2$$

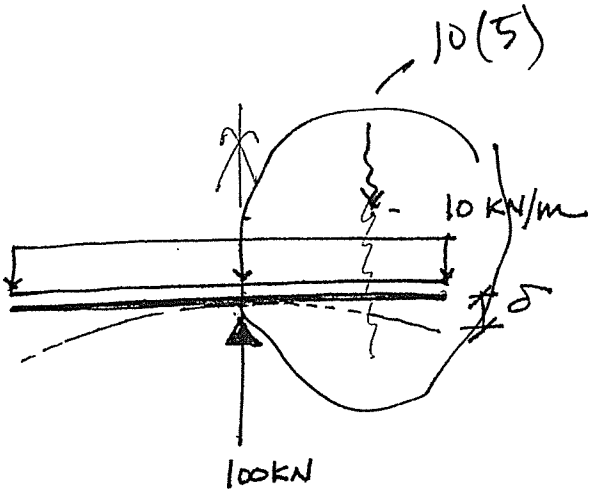
2º)

(25)





(25)

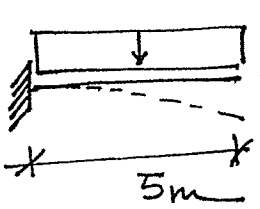


$V(x) = -10x$

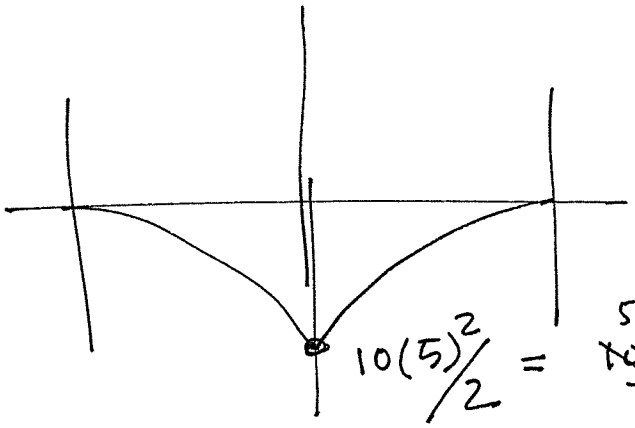
$M(x) = -5x^2$

$EI\frac{d^2y}{dx^2} = -5x^2$ $\quad \frac{5}{3}x^3 + C_1x = dy/dx$

$y = \frac{5}{12}x^4 + C_1x^2/2 + C_0$



$\delta = \frac{wL^4}{8EI} = \frac{10(5)^4}{8EI} = \boxed{\frac{781.25}{EI} = \delta}$



$\frac{10(5)^2}{2} = \frac{10(25)}{2} = 125$