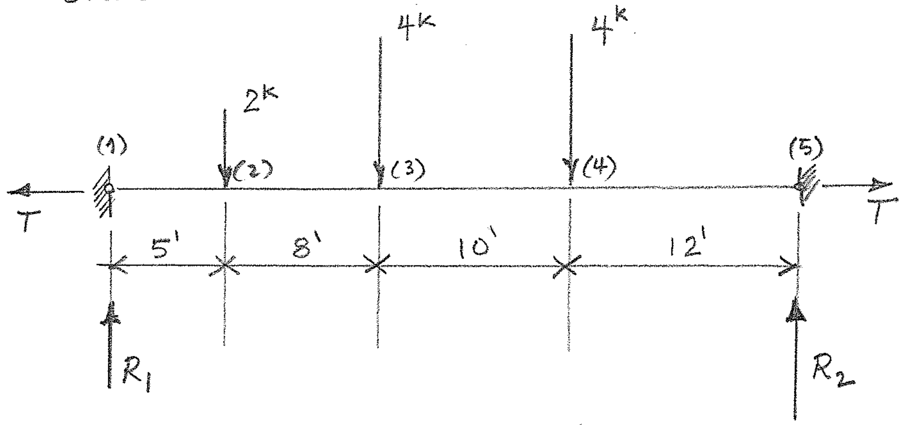
Determine the equilibrium position of the following cable structure



$T = 100^k$

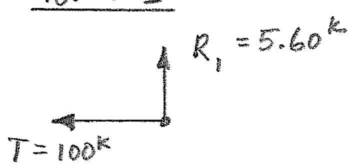
Reactions at both supports:

$$R_2 = \left[2(5) + 4(13) + 4(23) \right] / 35$$

$$R_2 = 4.40^k$$

$$R_1 = 2 + 4 + 4 + (-4.40) = 5.60^k$$

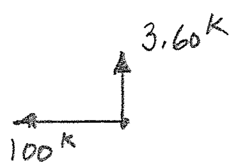
Point 1



$$\tan\theta_1 = \frac{5.60}{100} \Rightarrow \theta_1 = 3.205^\circ$$

$$h_2 = 5' \times 12 \times \tan(3.205) = 3.36''.$$

Point 2.



$$\tan\theta_2 = \frac{3.6}{100} \Rightarrow \theta_2 = 2.062^\circ$$

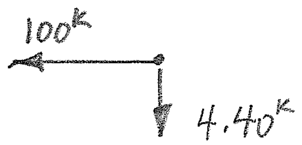
$$h_3 = h_2 + 8' \times 12 \times \tan(2.062^\circ) = 6.816''.$$

Point 3

$$\tan\theta_3 = \frac{-0.40}{100} \Rightarrow \theta_3 = -0.229^\circ$$

$$h_4 = h_3 + \tan\theta_3 \times 10' \times 12 = 6.336''.$$

Point 4.

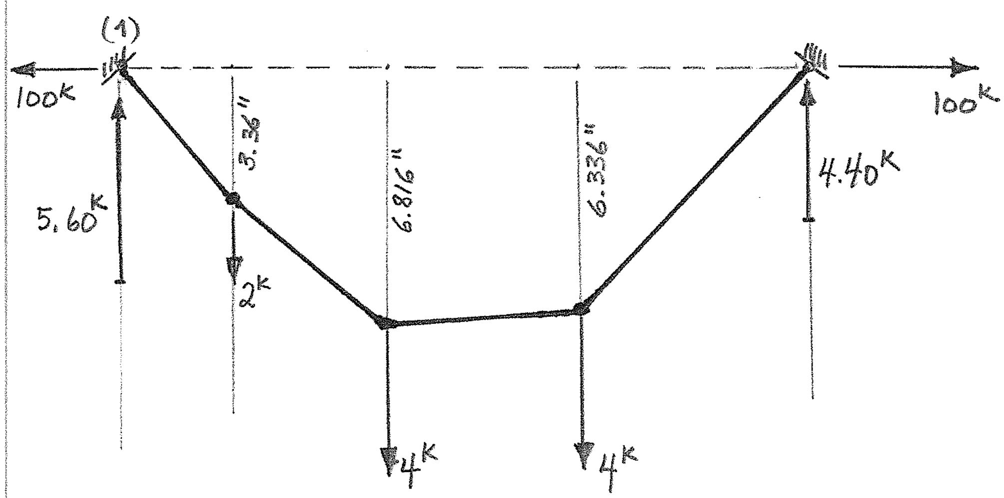


$$\tan \theta_4 = \frac{-4.40}{100} = -2.519^{\circ}$$

$$h_5 = h_4 + \tan \theta_4 \times 12' \times 12 = \underline{\underline{0}} \, .$$

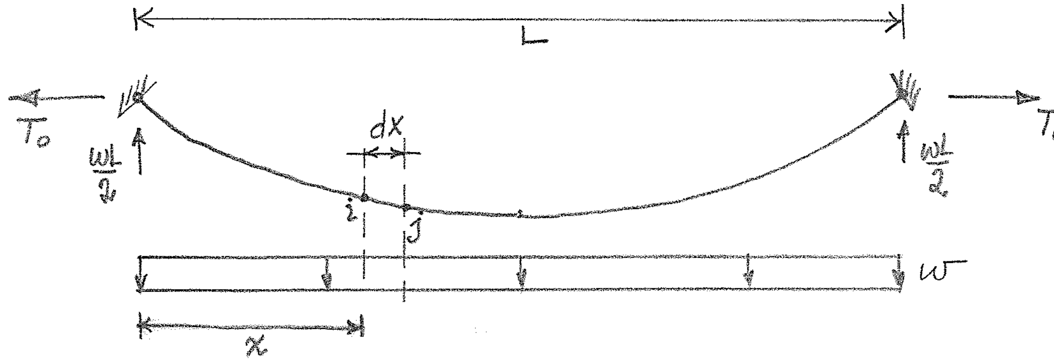
polygon closed!

Final Cable Configuration:

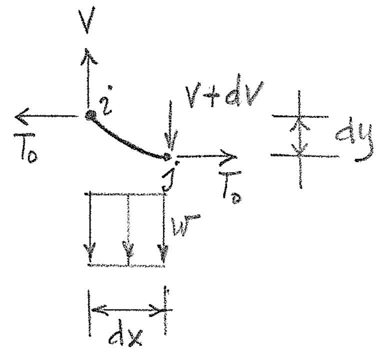


· Question: If the maximum cable deflection must be limited to 4"; what is the necessary pretension "T" of the cable?

• cable subject to uniform load (Suspension Cable)



neglect self weight.



$$\Sigma F_y = V - w\,dx - (V + dV) = 0$$

$$dV = -w\,dx$$

$$\boxed{\frac{dV}{dx} = -w}$$

$$\Sigma M_i = w\,dx \cdot \frac{dx}{2} + (V + dV)\,dx - T_o\,dy = 0$$

$$w\,dx \cdot \frac{dx}{2} + V\,dx - w\,dx \cdot dx - T_o\,dy = 0$$

$$\frac{w}{2}dx + V - w\,dx - T_o\frac{dy}{dx} = 0$$

$$-\frac{w}{2}dx - T_o\frac{dy}{dx} + V = 0 \qquad V = \frac{wL}{2} - wx$$

$$-\frac{w}{2}dx - T_o\frac{dy}{dx} + \frac{wL}{2} - wx = 0$$

→ this term can be neglected since it is a differential among non-differential terms.

$$-T_o\frac{dy}{dx} + \frac{wL}{2} - wx = 0 \qquad \text{(Now take derivatives w.r.t. } x\text{)}$$

$$\boxed{T_o\frac{d^2y}{dx^2} = w} \Rightarrow \frac{d^2y}{dx^2} = \frac{w}{T_o} \Rightarrow \boxed{y = \frac{w}{2T_o}x^2 + C_o x + C_1}$$

at $x = 0$ $y = 0$ then $\boxed{C_1 = 0}$

at $x = L$ $y = 0$ then $0 = \frac{wL^2}{2T_0} = -C_0 L \Rightarrow \boxed{C_0 = -\frac{wL}{2T_0}}$

then:

$$\boxed{y = \frac{w}{2T_0}x^2 - \frac{wL}{2T_0}x}$$

if the origin is chosen at the center then

$$x' = x - \frac{L}{2}$$

$$y = \frac{w}{2T_0}\left(x' + \frac{L}{2}\right)^2 - \frac{wL}{2T_0}\left(x' + \frac{L}{2}\right)$$

$$y = \frac{w}{2T_0}\left(x'^2 + x'L + \frac{L^2}{4}\right) - \frac{wL}{2T_0}x' + \frac{wL^2}{4T_0}$$

$$y = \frac{wx'^2}{2T_0} + \cancel{\frac{wx'L}{2T_0}} + \frac{wL^2}{8T_0} - \cancel{\frac{wLx'}{2T_0}} - \frac{wL^2}{4T_0}$$

$$\boxed{y = \frac{wx^2}{2T_0} - \frac{wL^2}{8T_0}}$$ where x is measured from the center.

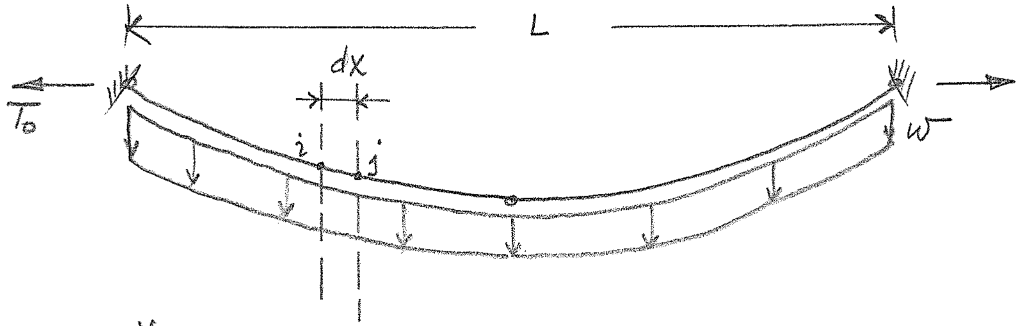
if cable height is measured from the bottom the eq. simplifies to:

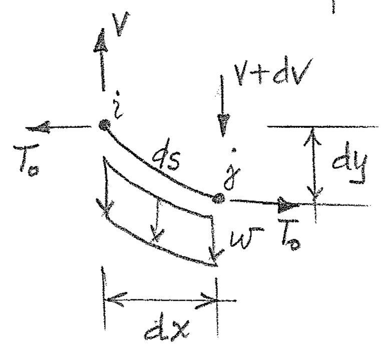
$$y = \frac{wx^2}{2T_0}$$

The max. tension in the cable is at the support and it is

$$T_{max} = \sqrt{T_0^2 + \left(\frac{wL}{2}\right)^2}$$

Cable subject to self-weight - CATENARY





$$\sum F_y = V - w\,ds - (V + dV) = 0$$

$$dV = -w\,ds$$

$$\boxed{\frac{dV}{ds} = w}$$

$$\sum M_i = w\,ds \cdot \frac{dx}{2} + (V + dV)\,dx - T_o\,dy = 0$$

$$w\,ds\,\frac{dx}{2} + V\,dx - w\,ds\,dx - T_o\,dy = 0$$

$$-w\,ds\,\frac{dx}{2} + V\,dx - T_o\,dy = 0$$

$$-\frac{w}{2}\,ds + V - T_o\frac{dy}{dx} = 0 \qquad V = \frac{wS}{2} - ws$$

S = total length of cable
$S \neq L$

$$-\frac{w}{2}\,ds + \frac{wS}{2} - ws - T_o\frac{dy}{dx}$$

↳ This term can be neglected since it is a differential among non-differentials.

$$\frac{wS}{2} - ws - T_o\frac{dy}{dx} = 0 \qquad \text{(taking derivative w.r.t } x\text{)}$$

$$\boxed{-w\frac{ds}{dx} = T_o\frac{d^2y}{dx^2}}$$

Note : $ds^2 = dx^2 + dy^2 \Rightarrow ds = \sqrt{dx^2+dy^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

calling $g = \frac{dy}{dx}$ $\quad ds = \sqrt{1+g^2}\ dx$

then we have :

$$-w\sqrt{1+g^2}\,\frac{\cancel{dx}}{\cancel{dx}} = T_0 \frac{dg}{dx}$$

$$-\frac{w}{T_0}dx = \frac{dg}{\sqrt{1+g^2}}$$ integrating both sides

$$-\frac{w}{T_0}x = \sinh^{-1} g$$

$$\sinh\left(-\frac{w}{T_0}x\right) = g$$

$$-\sinh\left(\frac{w}{T_0}x\right) = \frac{dy}{dx}$$ $\Rightarrow$ integrating both sides from 0 to x

$$\boxed{y = \frac{T_0}{w}\left(\cosh \frac{wx}{T_0} - 1\right)}$$ where x is measured from the center.

and cable height is measured from the bottom.

The length of the cable is :

$$S = \int_0^L \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^L \sqrt{1+\left(\sinh \frac{w}{T_0}x\right)^2}\,dx$$ $\Rightarrow$ note $\left(\sinh \frac{w}{T_0}x\right)^2 = \frac{1}{2}\cosh 2x - \frac{1}{2}$

$$S = \int_0^L \sqrt{\frac{1}{2} + \frac{1}{2}\cosh \frac{2wx}{T_0}}\,dx$$