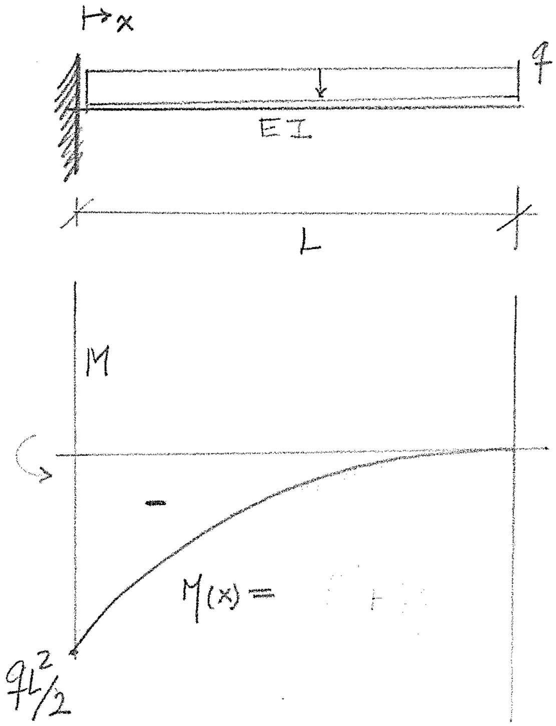
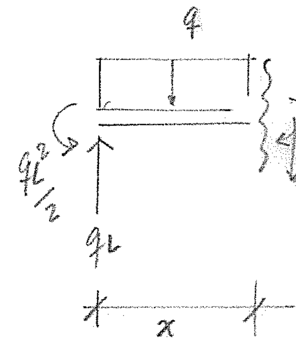




$V(x) = qL - qx$

$M = q(L-x)$

$M(x) = -qL^2/2 + qLx - qx^2/2$

$$EI \cdot \frac{d^2y}{dx^2} = M = -qL^2/2 + qLx - qx^2/2$$

$$EI \frac{dy}{dx} = -\frac{qL^2}{2} \cdot x + qL\frac{x^2}{2} - q\frac{x^3}{6} + c_0$$

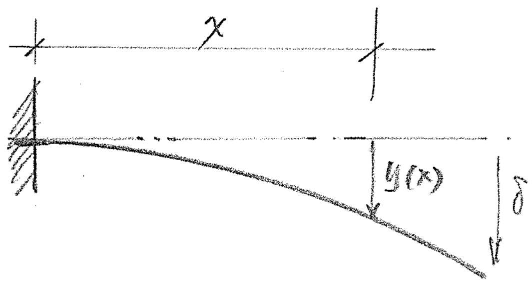
$$EI\, y = -\frac{qL^2}{4}x^2 + qL\frac{x^3}{6} - \frac{qx^4}{24} + c_0 x + c_1$$

B.C.

@ $x=0$ $y=0$ $\Rightarrow c_1 = 0$

@ $x=0$ $\frac{dy}{dx} = 0$ $\Rightarrow c_0 = 0$

$$y = \frac{1}{EI}\left(-\frac{qx^4}{24} + qL\frac{x^3}{6} - qL^2\frac{x^2}{4}\right)$$ deformed shape.



$$y(x) = \frac{1}{EI}\left(-\frac{qx^4}{24} + qL\frac{x^3}{6} - qL^2\frac{x^2}{4}\right)$$

y_{max} @ $x = L$

$$y_{max} = \delta = \frac{1}{EI}\left(-\frac{qL^4}{24} + \frac{qL^4}{6} - \frac{qL^4}{4}\right)$$

$$\delta = -\frac{1}{EI}\cdot\frac{3qL^4}{24} = \boxed{-\frac{qL^4}{8EI} = \delta}$$

$$M_{MAX} = \frac{qL^2}{2}$$

$$\delta = \frac{L^2}{4}\cdot\frac{M_{MAX}}{EI}$$

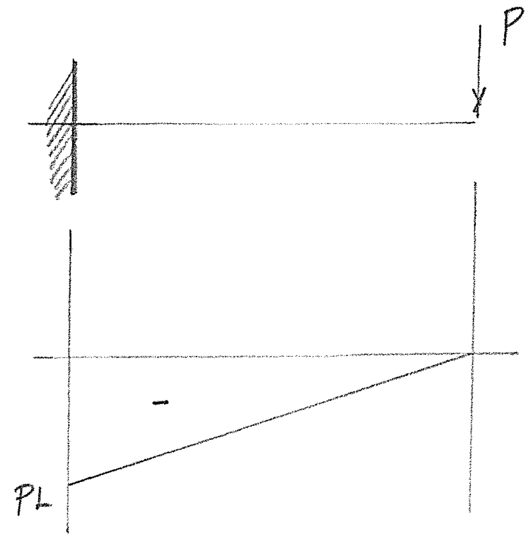
Typical problems:

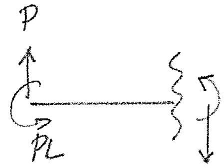
- ANALYSIS : Given q, L, E, I compute δ
- DESIGN : Given q, L, E find I to achieve a δ
- ESTIMATION : Given δ, q, L, I estimate E. or Given q, L estimate EI

  Given δ, L, E, I estimate q

  Given L, $y(x_1)$ and $y(x_2)$ estimate EI and q.

inverse problems!





$$M(x) = -PL + PX$$

$$EI \frac{dy^2}{dx^2} = M = PX - PL$$

$$EI \frac{dy}{dx} = \frac{PX^2}{2} - PL \cdot X + C_0$$

$$EI\, y = \frac{PX^3}{6} - PL \cdot \frac{X^2}{2} + C_0 X + C_1$$

B.C.

@ $X = 0$ $\quad y = 0$ & $dy/dx = 0$

$C_0 = 0$ & $C_1 = 0$

$$EI\, y(x) = \frac{PX^3}{6} - \frac{PL}{2} x^2$$

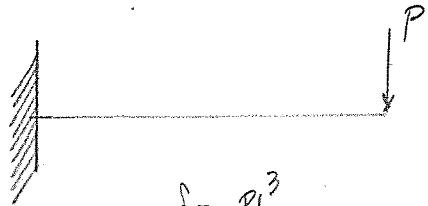
$$M_{max} = PL$$

$$EI\,\delta = \frac{PL^3}{6} - \frac{PL^3}{2} \Rightarrow \boxed{\delta = -\frac{PL^3}{3EI}}$$

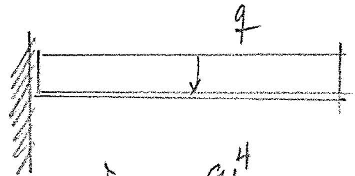
$$\delta = \frac{L^2}{3} \cdot \frac{M_{max}}{EI}$$

$$\delta = \frac{L^2}{3} \cdot K_{max}$$

<u>Comparison:</u>



$$\delta_p = \frac{PL^3}{3EI}$$



$$\delta_q = \frac{qL^4}{8EI}$$

if $P = q \cdot L$ then

$$\delta_p = \frac{qL^4}{3EI}$$

$$\frac{\delta_p}{\delta_q} = \frac{8}{3} = 2.67\ !$$