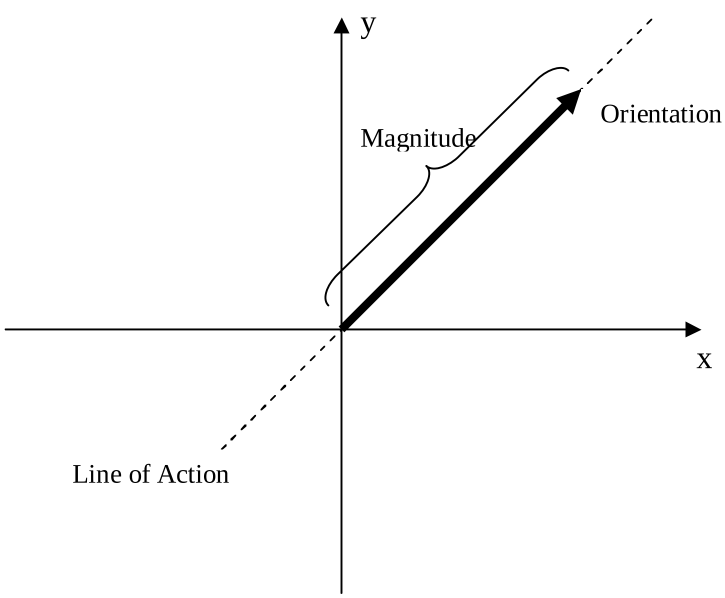
Structural Analysis – CE170 Instructor: Eric M. Hernandez, Ph.D.

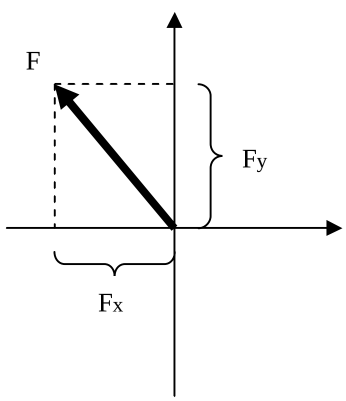
Theory of Forces

Definitions

According to Newton a force is the cause of change of momentum in a material point: $F = \frac{d}{dt}(mv)$, where m is the mass of the material point and v its velocity. In structures a force is also the cause of deformations. A force is clearly defined by three quantities; line of action, magnitude, and orientation, therefore forces are vectorial quantities.

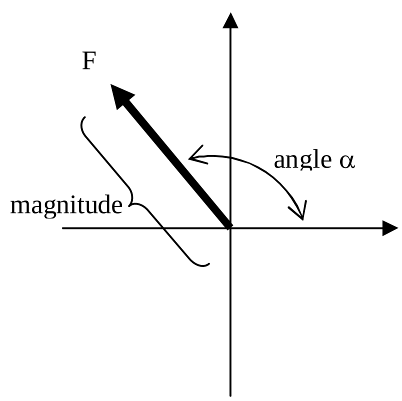


The line of action defines the geometrical position of the force; the magnitude of the force is represented graphically by the length of the line and the orientation by the direction of the arrow. There are fundamentally two ways of representing a vector, using rectangular coordinates or polar coordinates.



Cartesian coordinates

$$F = \sqrt{F_x^2 + F_y^2}$$



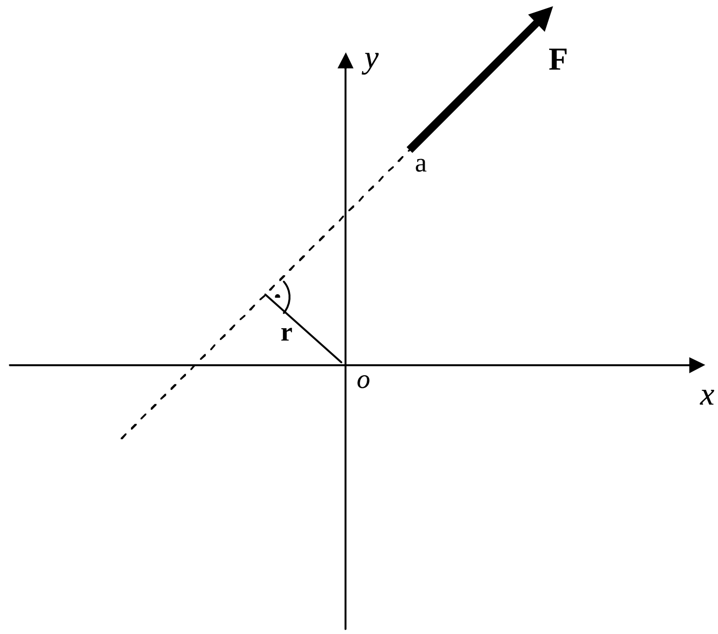
Polar coordinates

$$F_x = |F|\cos\alpha \quad F_y = |F|\sin\alpha$$

Moment of a Force

The moment (**M**) of a force with respect to a given point is defined as the rotational effect of a force on that point or equivalently, the change in the angular momentum with respect to a given point. The magnitude of the moment is equal to the product of the magnitude of the force (**F**) times the distance from the point perpendicular to the line of action of the force (**r**). (see figure below)

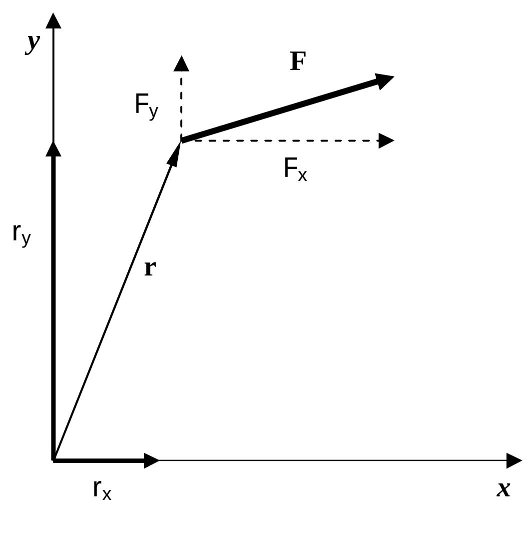
$$M_o = F\ r$$



For notational purposes the point where the moment is calculated is used a subscript on M, and it can be clearly seen that the force F does not produce any moment on a point located on its line of action.

Another procedure for determining the moment produced by a force is decomposing the force into its *x* and *y* components, and apply the concept previously explained to each of the components, see figure below. It is customary to assume that a moment that acts counter clockwise is positive and clockwise negative.

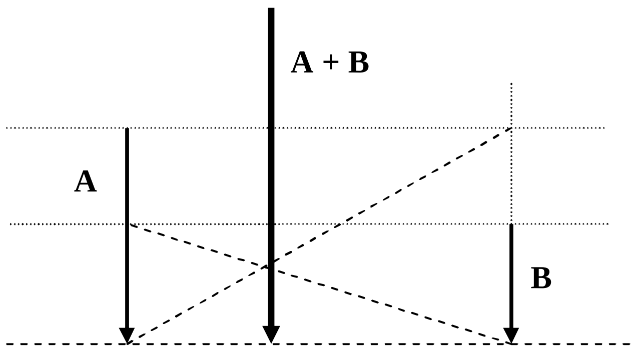
$$M = r_y F_x - r_x F_y$$



- Parallel Forces

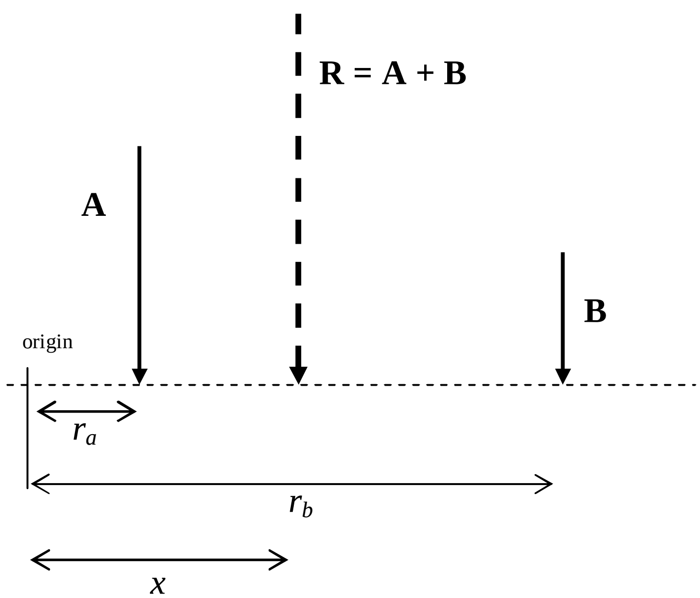
When the line of action of two forces does not meet we say that the forces are parallel. We can add parallel forces using numerical or graphical methods.

Graphical Method :



Note : Remember you must draw distances and forces to an appropriate scale.

Numerical Method :

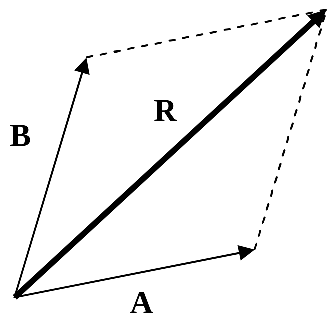


$R = A + B$ ....Resultant force R

$x = \frac{\sum M}{R} \frac{(A \times r_a) + (B \times r_b)}{(A + B)}$ ....Location of the resultant

- Concurrent Forces

Forces which have an intersecting line of action are concurrent. We can apply the parallelogram rule to graphically add the forces.



R = A + B

Numerically we can simply add the components to obtain the x and y component of the resultant:

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y$$

The magnitude of the resultant is simply $R = \sqrt{R_x^2 + R_y^2}$

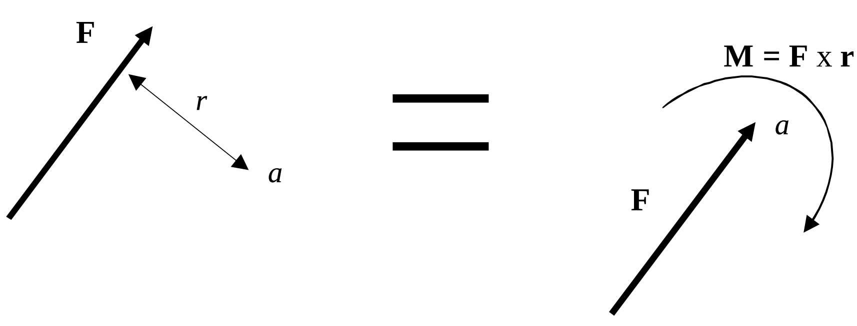
Any system of forces can be reduced to a single resultant force by successively adding each of the forces with the previous resultant. For example

$\mathbf{F_1 + F_2 = R_{12}}$
$\mathbf{F_3 + R_{12} = R_{123}}$
etc....

- The effect of a force on a give point

A force produces movement, and this movement can be linear or rotational. The linear effect is the force itself accelerating the mass on it's line of action, and the rotational effect is the moment of that force with respect to the point where the effect is investigated.

The figure above shows the force F and a point *a*, the effect of **F** on *a* is the force itself and a moment equal to the magnitude of the force time the distance from the point perpendicular to the line of action of the force.



Therefore we can conclude that the effect of any system of forces on a given point is reducible to one force and one moment, these produce a linear displacement and a rotational effect respectively.

- Concept of Equilibrium

A system of forces is in equilibrium when both of the following conditions are met:

$$\sum F = 0$$

$$\sum M = 0$$

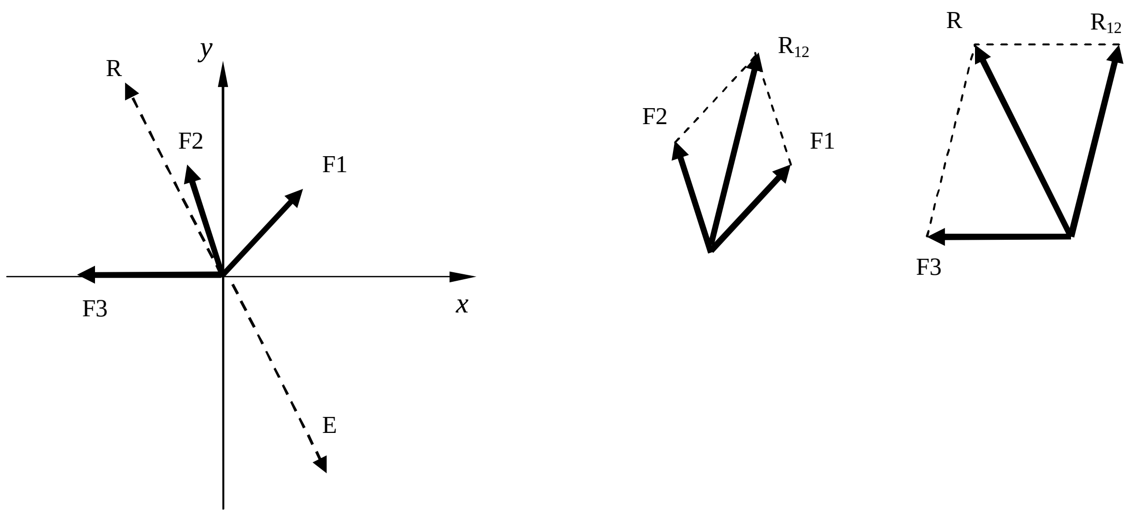
Thus a given group of forces with resultant **R** is equilibrated by another group of forces which has resultant **(–R)**. The vector **(–R)** is called the equilibrant **E**.

- Example

Given the following system of forces acting on the origin determine the resultant and the moment with respect to point 'a' located at (0,5)

Force	x-component	y-component
F1	3	3
F2	-2	4
F3	-5	0

GRAPHICAL SOLUTION



ANALYTICAL SOLUTION

Rx = 3 – 2 – 5 = - 4 x-component of resultant
Ry = 3 + 4 + 0 = 7 y-component of resultant

Magnitude of resultant = $\sqrt{(-4)^2 + (7)^2} = 8.06$
Angle of the resultant with respect ot the x-axis = atan(-4/7) + 90 = 119.75º

The moment produced by the resultant force around point a (0,5) is : M = (7)x(0) - (-5)x(-4) = -20