Structural Analysis and Design

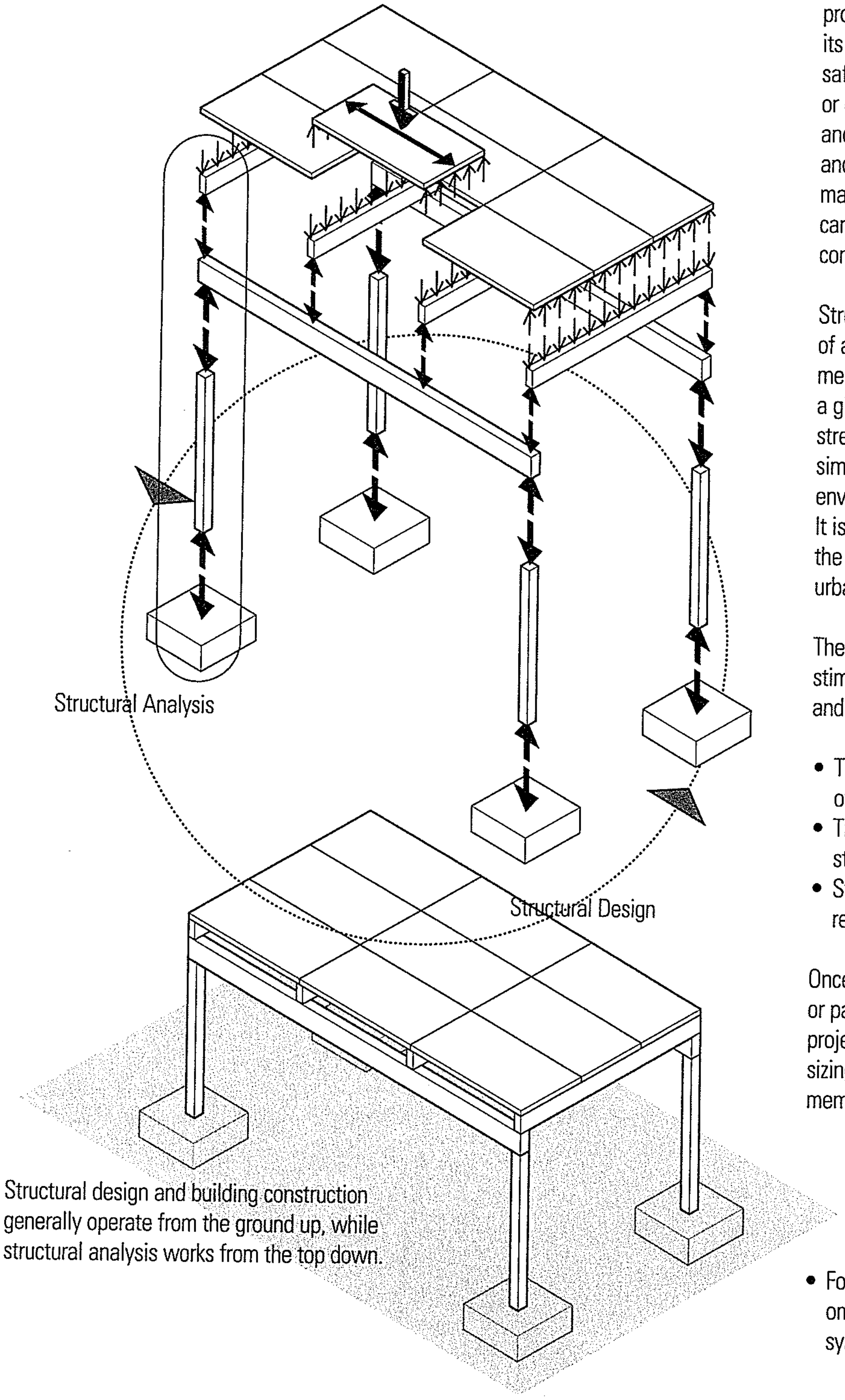
Before proceeding to a discussion of structural design, it might be useful to distinguish between structural design and structural analysis. Structural analysis is the process of determining the ability of a structure or any of its constituent members, either existing or assumed, to safely carry a given set of loads without material distress or excessive deformation, given the arrangement, shape, and dimensions of the members, the types of connections and supports utilized, and the allowable stresses of the materials employed. In other words, structural analysis can occur only if given a specific structure and certain load conditions.

Structural design, on the other hand, refers to the process of arranging, interconnecting, sizing, and proportioning the members of a structural system in order to safely carry a given set of loads without exceeding the allowable stresses of the materials employed. Structural design, similar to other design activities, must operate in an environment of uncertainty, ambiguity, and approximation. It is a search for a structural system that can meet not only the load requirements but also address the architectural, urban design, and programmatic issues at hand.

The first step in the structural design process may be stimulated by the nature of the architectural design, its site and context, or the availability of certain materials.

- The architectural design idea may elicit a specific type of configuration or pattern.
- The site and context may suggest a certain type of structural response.
- Structural materials may be dictated by building code requirements, supply, availability of labor, or costs.

Once the type of structural system, its configuration or pattern, and the palette of structural materials are projected, then the design process can proceed to the sizing and proportioning of assemblies and individual members and the detailing of connections.



Structural design and building construction generally operate from the ground up, while structural analysis works from the top down.

- For clarity, lateral-force-resisting elements have been omitted. See Chapter 5 for lateral-force-resisting systems and strategies.

In planning any structural system, there are two attributes that should be built into the design, guide its development, and ensure its stability, durability, and efficiency. These attributes—redundancy and continuity—apply not to a specific material or to an individual type of structural member, such as a beam, column, or truss, but rather to a building structure viewed as a holistic system of interrelated parts.

The failure of a building structure can result from any fracturing, buckling, or plastic deformation that renders a structural assembly, element, or joint incapable of sustaining the load-carrying function for which it was designed. To avoid failure, structural designs typically employ a factor of safety, expressed as the ratio of the maximum stress that a structural member can withstand to the maximum stress allowed for it in the use for which it is designed.

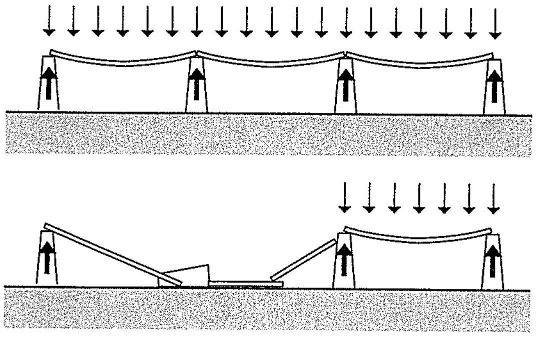
Under normal conditions, any structural element experiences elastic deformation—deflection or torsion—as a force is applied and as it returns to its original shape when the force is removed. However, extreme forces, such as those generated during an earthquake, can generate inelastic deformation in which the element is unable to return to its original shape. To resist such extreme forces, elements should be constructed of ductile materials.

Ductility is the property of a material that enables it to undergo plastic deformation after being stressed beyond the elastic limit and before rupturing. Ductility is a desirable property of a structural material, since plastic behavior is an indicator of reserve strength and can often serve as a visual warning of impending failure. Further, the ductility of a structural member allows excessive loads to be distributed to other members, or to other parts of the same member.

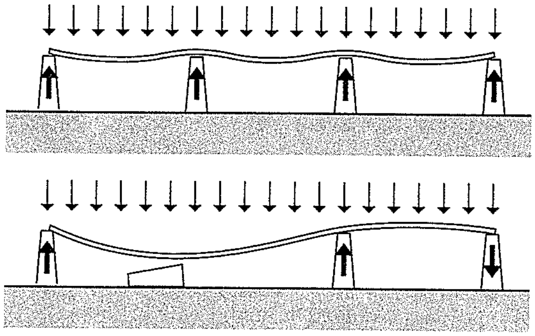
Redundancy

In addition to using factors of safety and employing ductile materials, another method for guarding against structural failure is to build redundancy into the structural design. A redundant structure includes members, connections, or supports not required for a statically determinate structure so that if one member, connection, or support fails, others exist to provide alternative paths for the transfer of forces. In other words, the concept of redundancy involves providing multiple load paths whereby forces can bypass a point of structural distress or a localized structural failure.

Redundancy, especially in the lateral-force-resisting systems of a structure, is highly desirable in earthquake-prone regions. It is also an essential attribute of long-span structures in which the failure of a primary truss, arch, or girder could lead to a large portion of the structure failing or even to its total collapse.

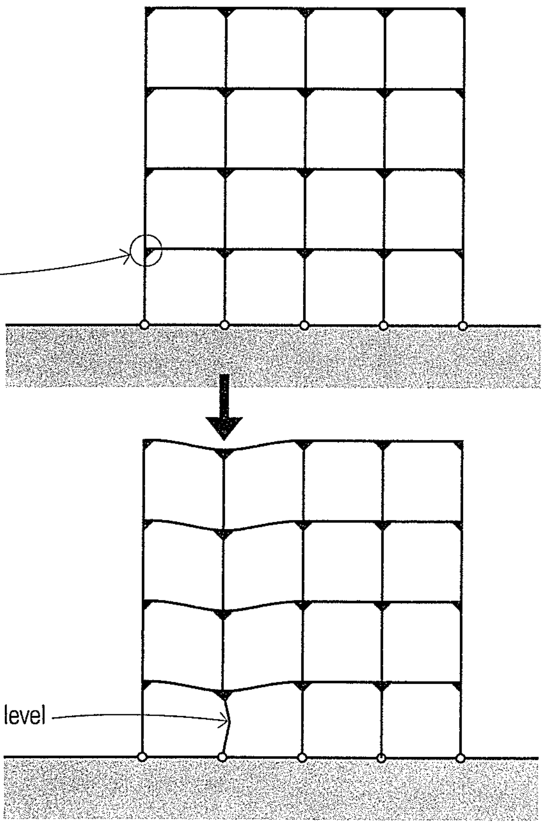


- Simple beams supported at their ends are determinate structures; their support reactions are easily determined through the use of the equations of equilibrium.



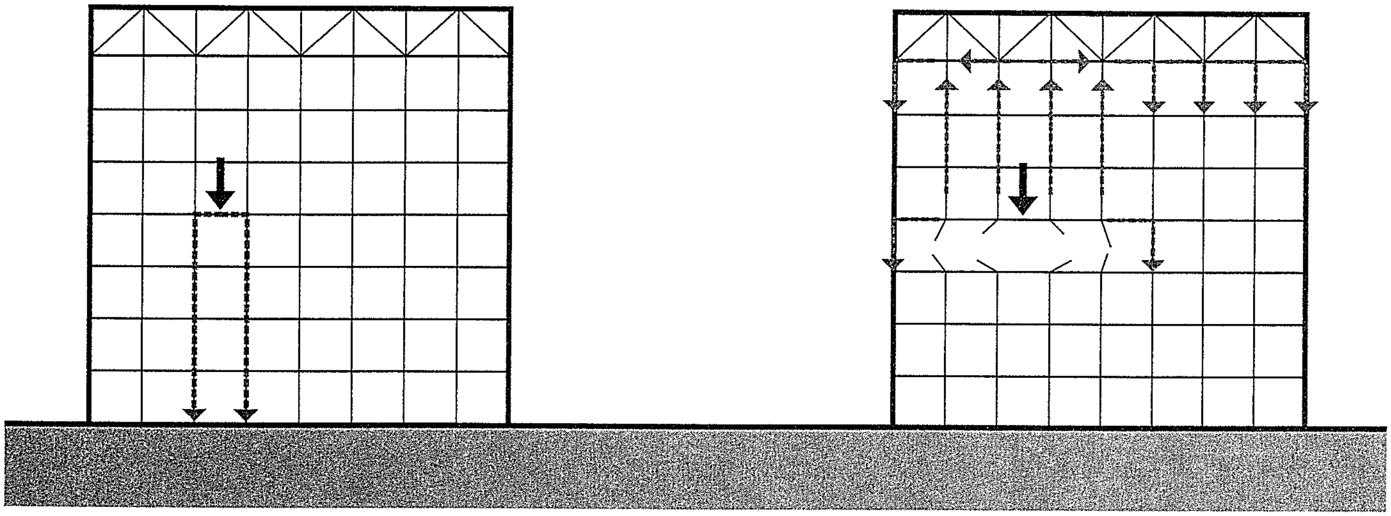
- If the same beam is continuous over four columns along its length, the structural assembly is indeterminate because there are more support reactions than the applicable equations of equilibrium. In effect, the continuity of the beam across multiple supports results in redundant paths for vertical and lateral loads to follow to the support foundations.

Extending structural redundancy to an entire structural system provides protection against progressive collapse of the structure. Progressive collapse can be described as the spread of an initial local failure from one structural member to another, eventually resulting in the collapse of an entire structure or a disproportionately large part of it. This is a major concern because progressive collapse can result in significant structural damage and loss of life.



- Concrete or steel frame with rigid joints
- Buckled column at first floor level

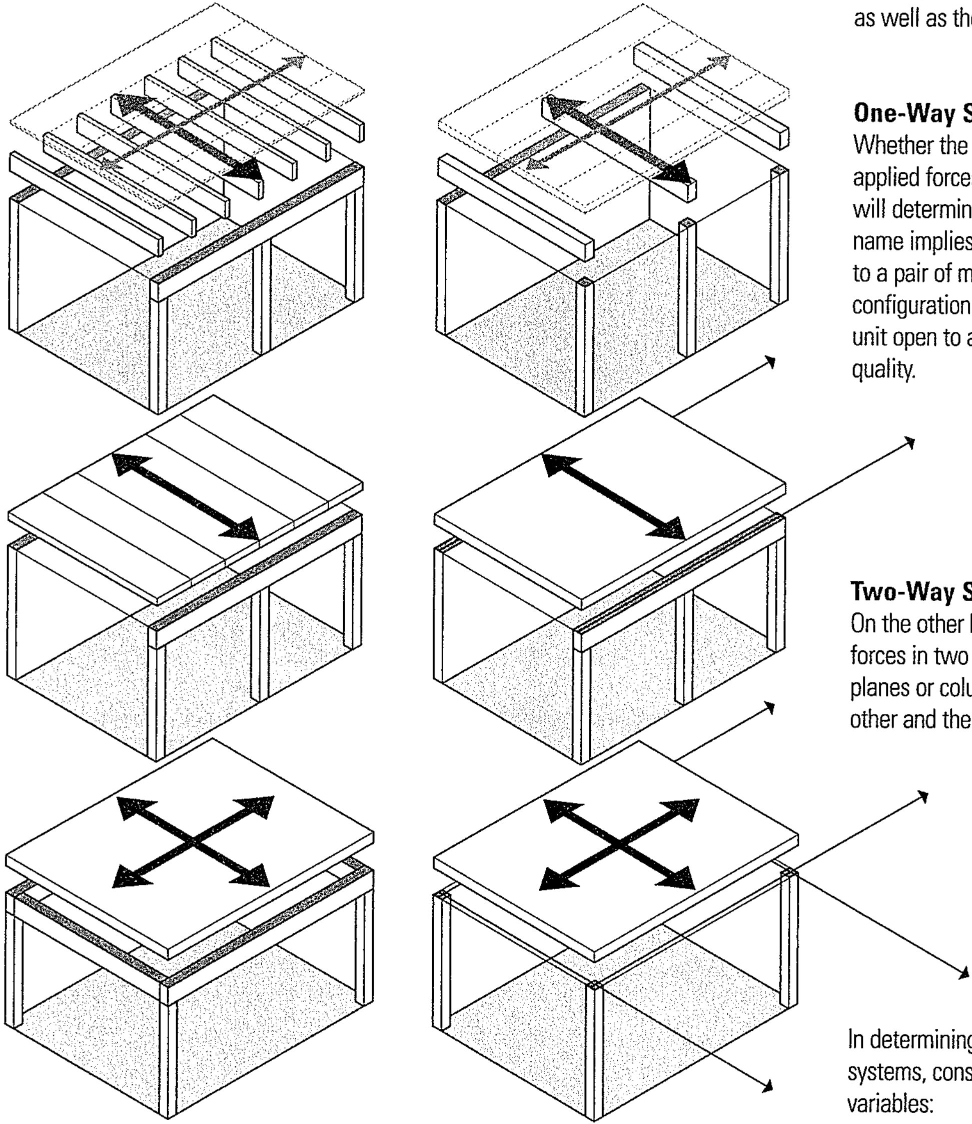
- A building frame connected with simple joints is subject to progressive collapse if one of its members or connections fails. With rigid beam-column connections, the same frame possesses ample load paths for both vertical and lateral loads.
- If a first-floor column were to fail, the rigid frame is able to redistribute the loads throughout the frame without collapsing.



- Vertical loads are normally received by a beam that redirects the load through bending to adjacent columns. The columns, in turn, transfer the loads in a continuous path down to the foundation.
- If columns at a particular story level are damaged or destroyed, the vertical loads are redirected by columns above to a major roof truss or girder. The truss or girder redistributes the loads to columns that are still functional. Redundancy in the overall building structure provides alternate load paths and helps prevent progressive collapse.

Spanning Options

Creating a spatial volume requires a minimum of two vertically oriented support planes, be they column-and-beam frames, bearing walls, or a combination thereof. To provide shelter against the vagaries of weather as well as a sense of enclosure, some sort of spanning system is required to bridge the space between the support systems. In looking at the fundamental ways of spanning the space between two support planes, we must consider both the way applied forces are distributed to the supporting planes as well as the form of the spanning system.



One-Way Spanning Systems

Whether the spanning system transfers and distributes applied forces in one or two (or even multiple) directions will determine the pattern of supports required. As the name implies, one-way systems transfer applied forces to a pair of more or less parallel supporting planes. This configuration naturally leaves two sides of the spatial unit open to adjacent spaces, giving it a strong directional quality.

Two-Way Spanning Systems

On the other hand, two-way systems transfer applied forces in two directions, requiring two sets of supporting planes or columns, more or less perpendicular to each other and the direction of transfer of forces.

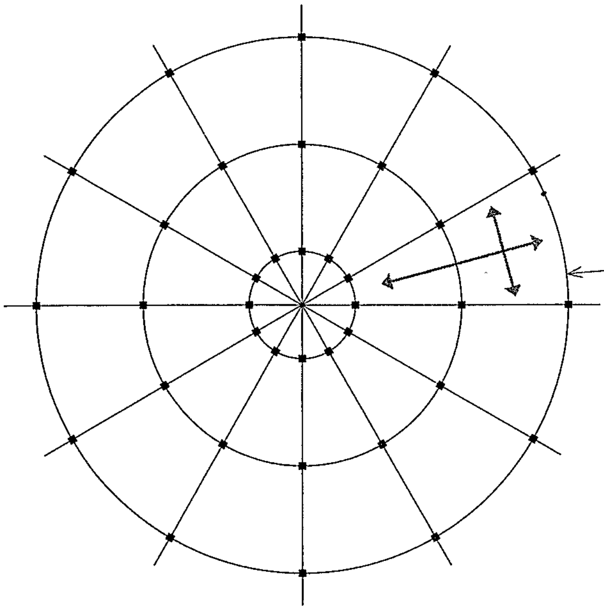
In determining whether to use one-way or two-way systems, consideration must be given to a number of variables:

- Dimensions, scale, and proportions of structural bay
- Structural materials employed
- Depth of construction assembly

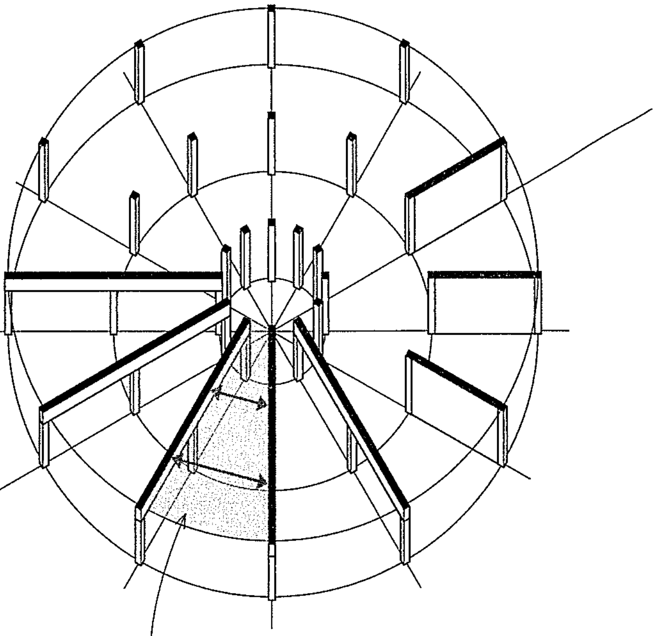
For more detailed information, see Chapters 3 and 4.

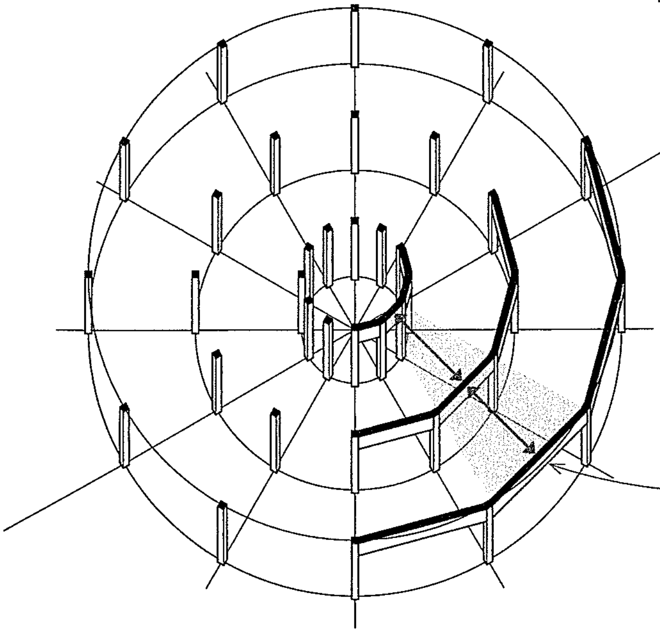
Radial Grids

Radial grids consist of vertical supports arranged in a radial pattern about a real or implied center. The direction of span is influenced by the support spacing, measured both radially and circumferentially.



- While one-way structures typically span the irregularly shaped bays, two-way flat plates or flat slabs can also span radial support patterns in an efficient manner.





- Collector beams or girders of constant length can span in a radial pattern, with feeder beams or joists having varying span lengths.
- If collector beams or girders span in a circumferential manner, their spans will vary while the feeder beams or joists will have constant span lengths.

- See pages 274–276 for dome structures.

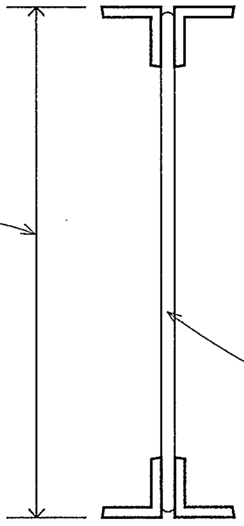
Open-Web Steel Joists

Open-web joists are lightweight, shop-fabricated steel members having a trussed web. They provide an economical alternative to steel beams for light to moderate distributed loads, especially for spans greater than 32' (10 m).

- K series joists have webs consisting of a single bent bar, running in a zigzag pattern between the upper and lower chords.
- 8" to 30" (205 to 760) depths

- LH (long-span) series joists and DLH (deep long-span) series joists have heavier web and chord members for increased loads and spans.
- LH series joist depths: 18" to 48" (455 to 1220)
- DLH series joist depths: 52" to 72" (1320 to 1830)

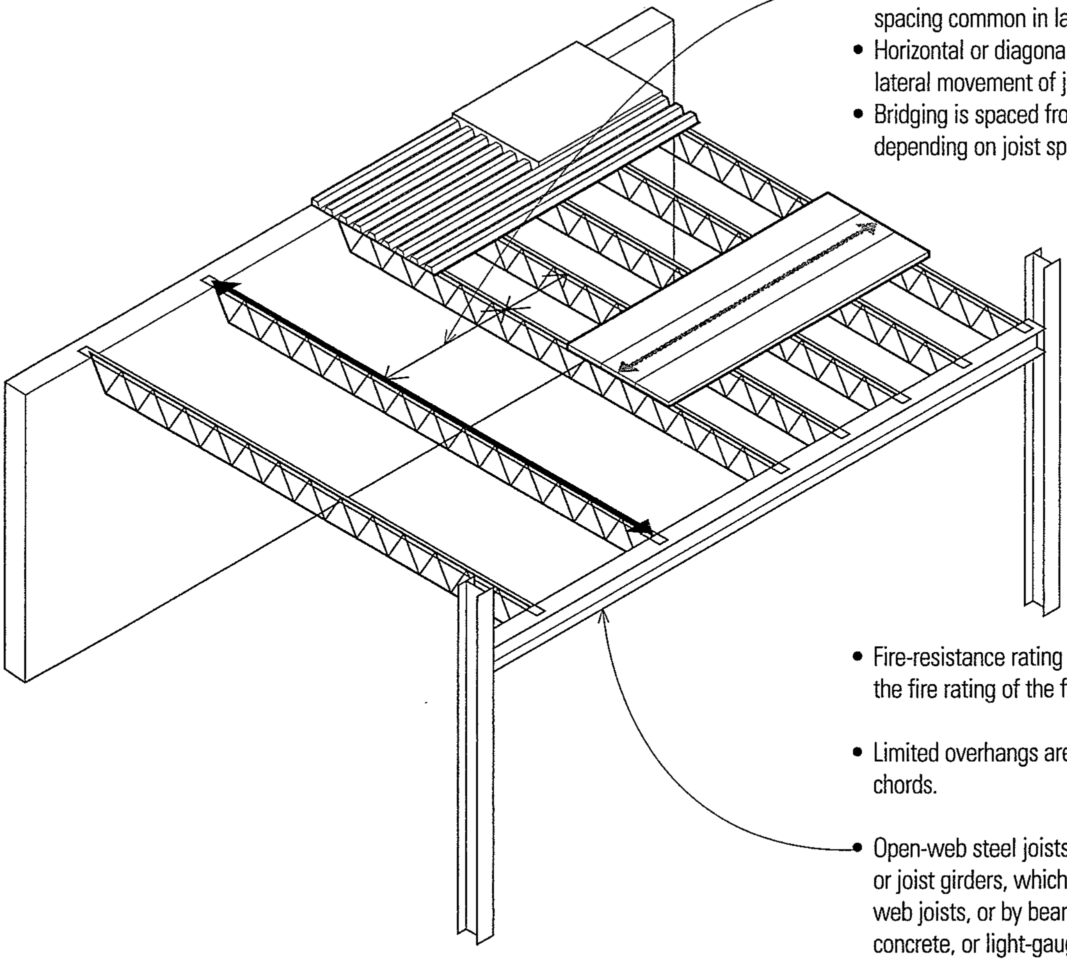
- Span range or open-web steel joists: 12' to 60' (3.6 to 18 m)
- Rule of thumb for estimating open-web joist depth: span/24



- The framing works most efficiently when the joists carry uniformly distributed loads. If properly engineered, concentrated loads may bear over the panel points of the joists.

- Open webs permit the passage of mechanical services.
- Ceiling may be attached to bottom chords or be suspended if additional space for services is required; ceiling may also be omitted to expose joists and floor deck.

- 2' to 10' (0.6 to 3 m) spacing; 4' to 8' (1.2 to 2.4 m) spacing common in large buildings
- Horizontal or diagonal bridging is required to prevent lateral movement of joist chords.
- Bridging is spaced from 10' to 20' (3 to 6 m) o.c., depending on joist span and chord size.



- Fire-resistance rating of the joist structure depends on the fire rating of the floor and ceiling assemblies.

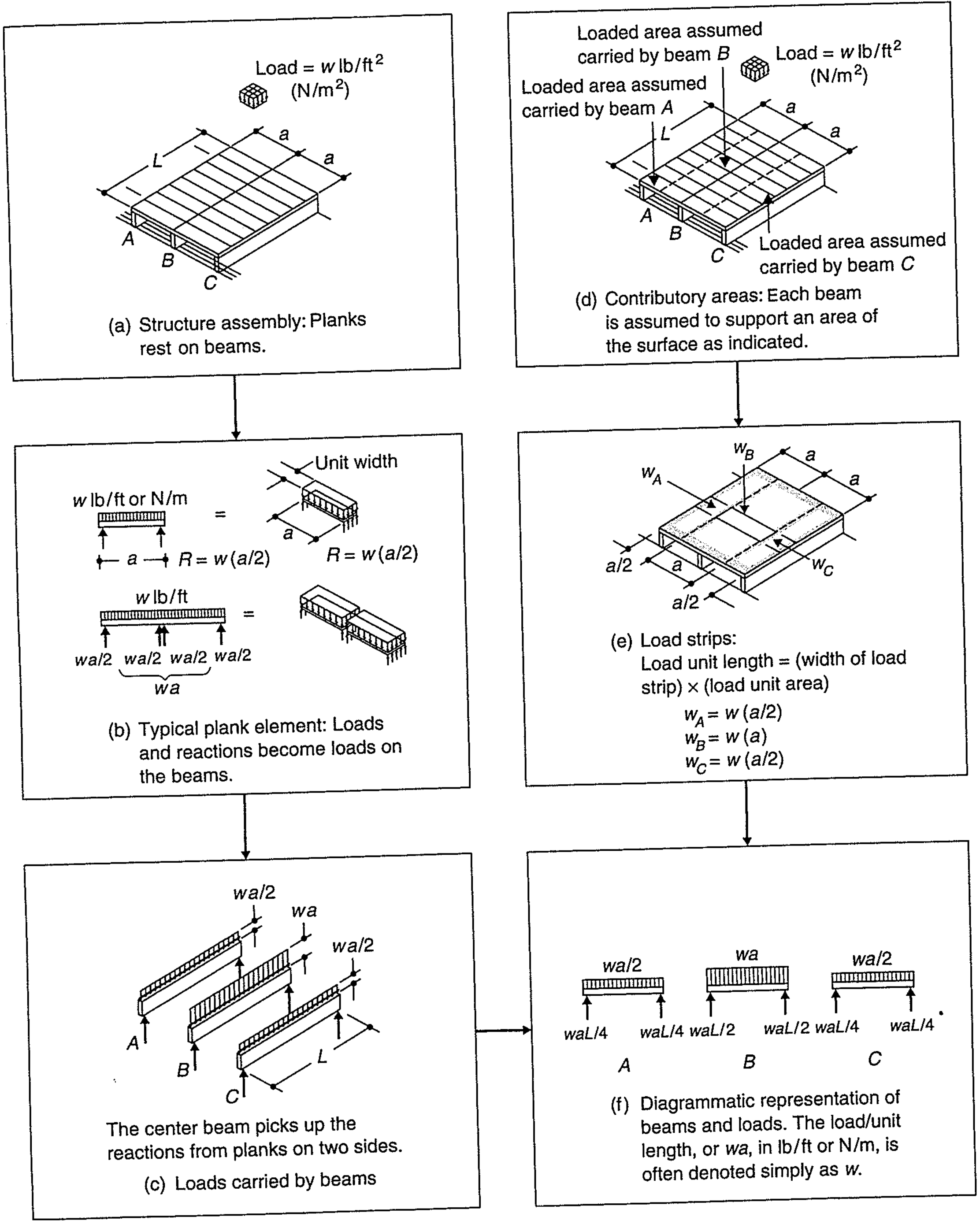
- Limited overhangs are possible by extending the top chords.

- Open-web steel joists may be supported by steel beams or joist girders, which are heavier versions of open-web joists, or by bearing walls of masonry, reinforced concrete, or light-gauge steel framing.

Floor System

Consider the simple structural assembly shown in Figure 3.13(a). Because all connections in this illustration are simply supported, the structure can be decomposed as indicated. For the wide plank elements, the reactions are better characterized as *line* reactions [as illustrated in Figure 3.13(b)] than as point reactions. The reactions from all the planks supported by a beam then become loads acting on the beam. Note that these loads form a continuous line load. Loads of this type are expressed in terms of a load or force per unit length (e.g., lb/ft or kN/m) and are

FIGURE 3.13 Two approaches to modeling loading conditions. The model shown on the left most correctly reflects how a beam system picks up surface loads. The model on the right, which is based on the concept of contributory areas, is often used for convenience. Both approaches yield the same loading on the beams.



(a) Structure assembly: Planks rest on beams.

(d) Contributory areas: Each beam is assumed to support an area of the surface as indicated.

(b) Typical plank element: Loads and reactions become loads on the beams.

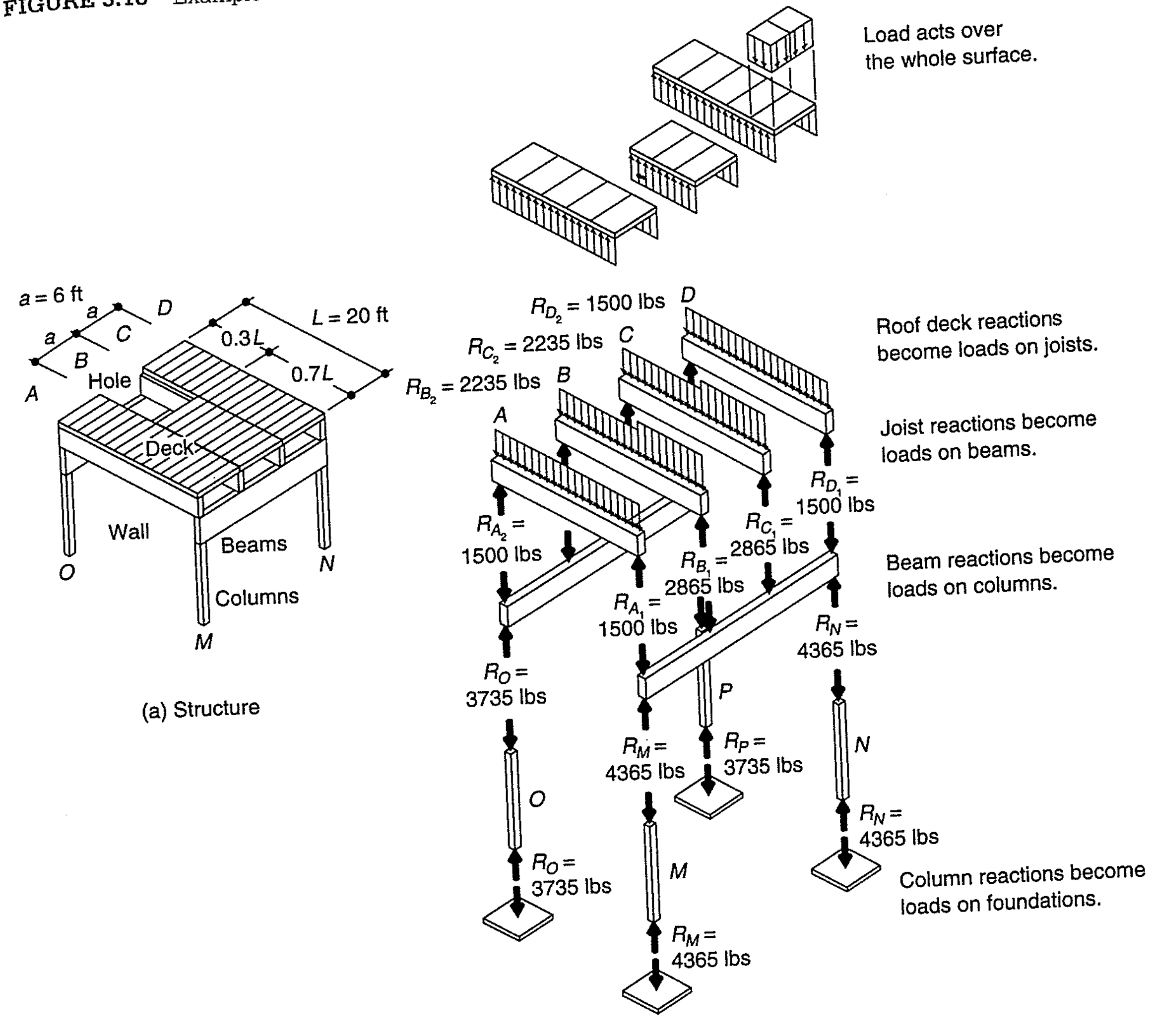
(e) Load strips:
Load unit length = (width of load strip) × (load unit area)
$w_A = w(a/2)$
$w_B = w(a)$
$w_C = w(a/2)$

The center beam picks up the reactions from planks on two sides.

(c) Loads carried by beams

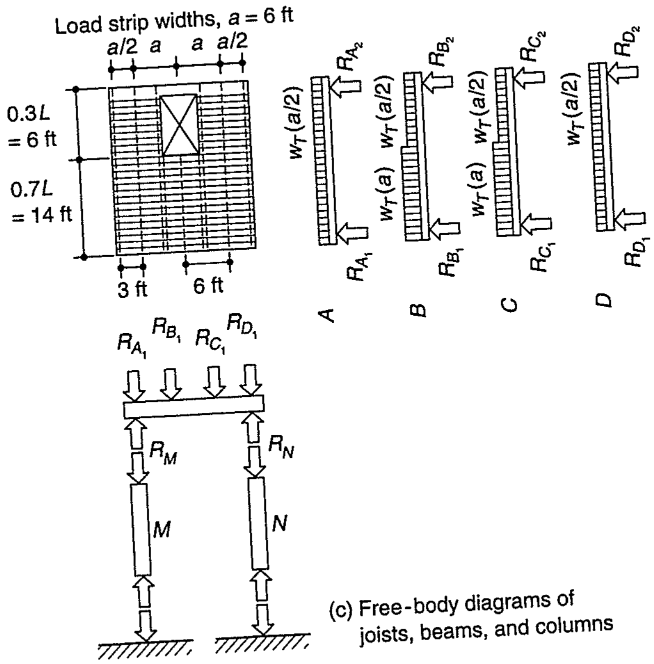
(f) Diagrammatic representation of beams and loads. The load/unit length, or wa, in lb/ft or N/m, is often denoted simply as w.

FIGURE 3.18 Example structure.



(a) Structure

(b) Free-body diagrams



(c) Free-body diagrams of joists, beams, and columns

Solution:

The first step is to determine how the surface load is channeled to the columns. This is best done by drawing free-body diagrams for each element in the structure. (See Figure 3.18.) To determine the magnitudes of the column loads, it is necessary first to calculate the load carried by each joist, then to calculate the reactions for each joist, and finally to calculate the reactions of the beams that carry the joists. The latter reactions are the column loads. The process of starting with an analysis of the smallest members picking up the loading and tracing the analysis through by considering each collecting member is common in most analytical problems.

Basic behavior

The decking transfers loads to the joists. The joist reactions become forces applied on the two transverse collector beams. The reactions of the two transverse beams become the forces exerted on the columns, which in turn become forces on the foundations.

Joist reactions: members A *and* D

The load per unit length carried by each joist is found by considering the width of the load strip carried by each joist. The end joists, *A* and *D*, carry load strips $a/2$ in width. If the total load per unit area is the w_T load per unit length carried by joists *A* and *D* is simply $w_T(a/2)$.

By symmetry,

$$R_{A1} = R_{A2} = w_T(a/2)(L) \div 2 \qquad \therefore \quad R_{A1} = R_{A2} = 0.25 w_T a L$$

$$R_{A1} = R_{A2} = 0.25(50)(6)(20) = 1500\text{ lb}$$

Or, by summing moments,

$\Sigma M_{RA1} = 0$:

$$-\underbrace{[w_T(a/2)(L)]}_{\text{total load}}\underbrace{(L/2)}_{\substack{\text{moment}\\ \text{arm}}} + \underbrace{1.0\,L}_{\substack{\text{moment}\\ \text{arm}}}\ \underbrace{(R_{A2})}_{\substack{\text{unknown}\\ \text{reaction}}} = 0 \qquad R_{A2} = 0.25 w_T a L$$

$$-[(50)(6/2)(20)](20/2) + 1.0(20)R_{A2} = 0 \quad R_{A2} = 1500\text{ lb}$$

$\Sigma F_y = 0$:

$$\underbrace{+R_{A1}}_{\substack{\text{unknown}\\ \text{reaction}}} + \underbrace{R_{A2}}_{\substack{\text{calculated}\\ \text{above}}} - \underbrace{w_T(a/2)(L)}_{\substack{\text{total}\\ \text{downward}\\ \text{load}}} = 0 \qquad R_{A1} = 0.25\,w_T a L$$

$$R_{A1} + 1500 - 50(6/2)(20) = 0 \qquad R_{A1} = 1500\text{ lb}$$

Similarly,

$$R_{D1} = 0.25 w_T a L = 1500\text{ lb} \quad \text{and} \quad R_{D2} = 0.25 w_T a L = 1500\text{ lb}$$

Joist reactions: members B and C

The load per unit length carried by joists *B* and *C* is not constant, due to the presence of the opening. In sections where the decking is continuous, the width of the load strip is *a*. The load per unit length is consequently $w_T(a)$. In sections where the hole is present, each joist carries a load strip of one-half that in continuous sections, or The load per unit length on these sections is $w_T(a/2)$.

$\Sigma M_{RB_1} = 0$:

$$-\underbrace{[w_T(a)(0.7\,L)]}_{\text{load}}\underbrace{(0.7\,L/2)}_{\substack{\text{moment}\\ \text{arm}}} - \underbrace{[w_T(a/2)(0.3\,L)]}_{\text{partial load}}\underbrace{(0.7\,L + 0.3L/2)}_{\substack{\text{moment}\\ \text{arm}}} + \underbrace{R_{B_2}}_{\text{reaction}}\ \underbrace{(1.0L)}_{\substack{\text{moment}\\ \text{arm}}}$$

$$-[50(6)(0.7)(20)](0.7)(20/2) - [50(6/2)(0.3)(20)][0.7(20)+0.3(20/2) + R_{B_2}(20) = 0$$

$$R_{B_2} = 0.3725 w_T a L = 2235\text{ lb}$$

$\Sigma F_y = 0$:

$$\underbrace{R_{B1}}_{\text{reaction}} + \underbrace{R_{B2}}_{\text{reaction}} - \underbrace{[w_T(a)(0.7L)]}_{\text{load}} - \underbrace{[w_T(a/2)(0.3L)]}_{\text{partial load}} = 0 \qquad R_{B1} = 0.4775 w_T a L$$

$$R_{B1} + 2235 - [50(6)(0.7)(20)] - [(50)(6/2)(0.3)(20)] = 0 \quad R_{B1} = 2865\text{ lb}$$

Similarly,

$$R_{C1} = 0.4775 w_T a L = 2865 \text{ lb} \quad \text{and} \quad R_{C2} = 0.3725 w_T a L = 2235 \text{ lb}$$

Column forces:
The forces on columns M and N are the reactions of the transverse beam-carrying joist loads (reactions): R_{A1}, R_{B1}, R_{C1}, and R_{D1}.

By symmetry,

$$\begin{aligned} R_M = R_N &= (R_{A1} + R_{B1} + R_{C1} + R_{D1}) \div 2 \\ &= (1500 + 2865 + 2865 + 1500)/2 \\ &= 4365 \text{ lb} \end{aligned}$$

or, by summing moments.

Beam $MN\ \Sigma M_{R_M} = 0$:

$$-R_{A1}(0) - R_{B1}(a) - R_{C1}(2a) - R_{D1}(3a) + R_M(0) + R_N(3a) = 0$$

$$-1500(0) - (2865)(6) - 2865(2)(6) - 1500(3)(6) + R_M(0) + R_N(3)(6) = 0$$

$$R_N = 0.7275 w_T a L = 4365 \text{ lb} \quad \text{and}$$

$$\Sigma F_y = 0: \quad -R_{A1} - R_{B1} - R_{C1} - R_{D1} + R_M + R_N = 0$$

$$R_M = 0.7275 w_T a L = 4365 \text{ lb}$$

The forces on columns O and P are the reactions of the second transverse beam.

By symmetry,

$$\begin{aligned} R_O = R_P &= (R_{A2} + R_{B2} + R_{C2} + R_{D2}) \div 2 \\ &= (1500 + 2235 + 2235 + 1500) \div 2 \\ &= 3735 \text{ lb} \end{aligned}$$

Equilibrium checks:
The total load that acts downward on the structure is $w_T \times A$, where A is the area of the surface. The forces developed on the four foundation points must sum to this same value (which provides a good overall check on your results):

$$R_M + R_N + R_O + R_P = w_T \times A$$

$$4365 + 4365 + 3735 + 3735 = 50 \text{ lb/ft}^2 (324 \text{ ft}^2)$$

$$16{,}200 \text{ lb} = 16{,}200 \text{ lb}$$

Alternatively, using metric units, assume that $L = 6.1$ m, $a = 1.83$ m, and $w_T = 2.394 \text{ kN/m}^2$. The force on column M is then given by $0.7275(2.39 \text{ kN/m}^2)(1.83 \text{ m})(6.1 \text{ m}) = 19.41$ kN.

EXAMPLE

Determine the forces on a typical interior truss in the structure illustrated in Figure 3.19. Also determine the reactions for the truss analyzed. Assume that $L_1 = 60 \text{ ft} = 18.3$ m and $L_2 = 25 \text{ ft} = 7.6$ m and that the loads are as shown in the following solution.

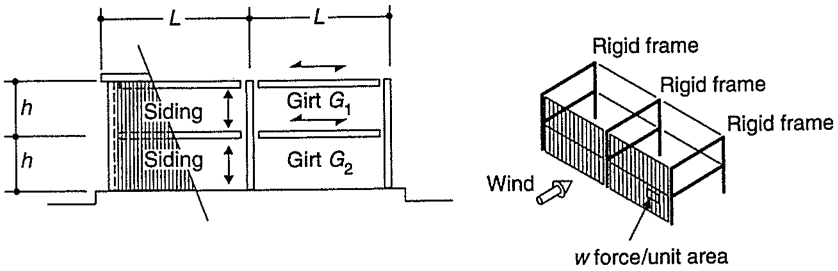
Solution:
Loads:
Assume that the live load is 35 lb/ft^2 (1.676 kN/m^2). Assume the following dead loads:

Roofing	7.0 lb/ft^2	$= 335 \text{ N/m}^2$
Sheathing	2.5	= 120
Joists and beams	8.0	= 383
Truss	4.0	= 191
Total	21.5 lb/ft^2	$= 1.029 \text{ kN/m}^2$

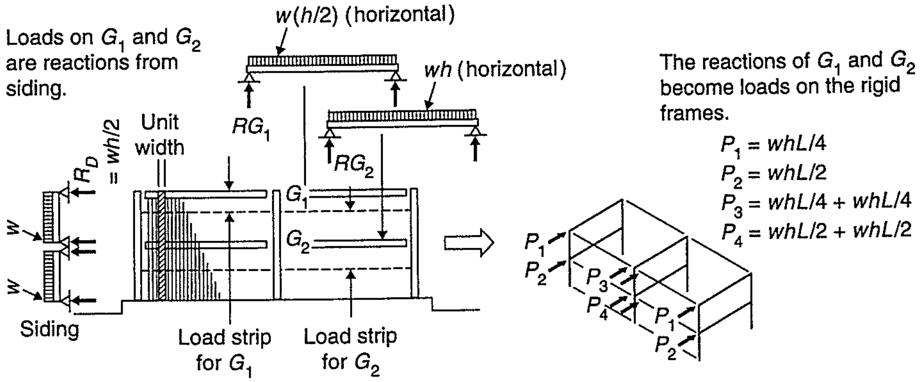
EXAMPLE

Determine the forces on the structural frames shown in Figure 3.21 due to the action of wind impinging on the face of the building. Assume that $w = 20\,\text{lb/ft}^2$, $h = 15$ ft, and $L = 25$ ft.

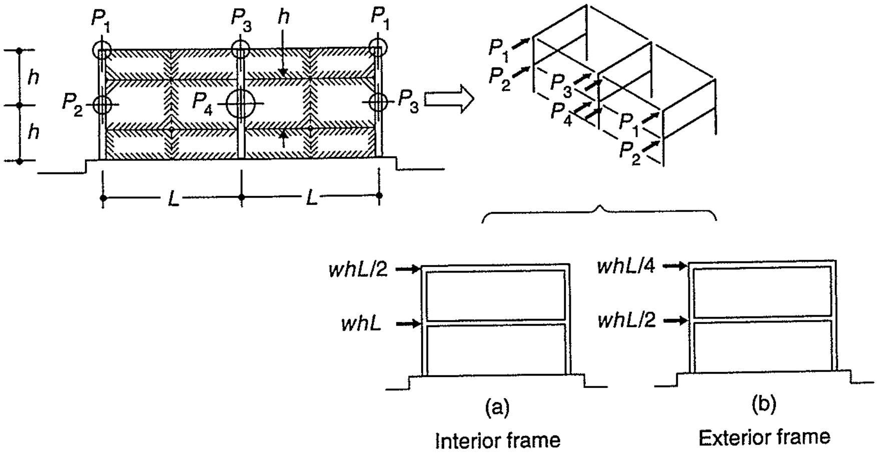
FIGURE 3.21 Modeling wind loads on rigid frames.



(a) Diagram of basic structural and enclosure system



(b) Model of how wind loads are transferred to rigid frames: The siding first picks up the distributed wind forces and transfers them to the horizontal girts (G_1 and G_2), which in turn transfer them to the frames.



(c) Simplified model for directly determining loads on the frames: The loads on the rigid frames can be determined directly by considering the contributory are carried by each load point on the rigid frames. If the enclosure framing were different, the way the load areas are assigned might also be different.

Solution:
The forces can be determined either by tracing how specific elements, such as girts, are loaded and how their reactions become loads on the frames or by using a contributory-area concept. Both methods are illustrated in Figure 3.13. Numerical values can be found by substitution (e.g., $whL/4$: 1875 lb).

QUESTIONS

3.1. Find two bridges in your area and identify the support conditions present in them (e.g., hinged, roller). Sketch the supports and include a diagram of the symbol that represents them.

3.2. Find two different trusses used in buildings in your area and identify the support conditions present in them. Sketch the end condition and include a diagram of the symbol that represents the support.

3.3. Consider a typical corridor in the building where you work or in one nearby. What do you estimate the *actual* live load to be during normal traffic conditions? During fire conditions?

3.4. What snow loads and wind loads are specifically recommended for buildings in your area? Consult the local building code.

3.5. For the floor system shown in Figure 3.13, assume that the combined live and dead load that is present is equal to 80 lb/ft^2. Assume a beam span of 16 ft and a beam spacing of 3 ft. Determine the reactions for beams A, B, and C in the floor system. (First, determine the load strip widths, then determine the appropriate loading model for each beam, and finally determine reactions for each beam by a statics analysis.)

Answer: Beam A: 960 lb each end; beam B: 1920 lb each end; beam C: 960 lb each end

3.6. Determine the reactions to Beam D in Figure 3.22. Assume that the average dead and live load is 60 lbs/ft^3.

Answers: 4896 lb, 4464 lb

3.7. Find the forces in Columns 1, 2, 3, and 4 of the structure shown in Figure 3.22. This is the same structure shown in Figures 3.1 and 3.15. Assume that the average dead load and live load is 60 lbs/ft^2.

Answers: Column 1, 8496 lb; Column 2, 8496 lb;
Column 3, 12,384 lb; Column 4, 12,384 lb

3.8. For the beam shown in Figure 3.21(a), determine the reactions if the live load is changed to 30 lb/ft. Repeat the example analyzed in Figure 3.21, assuming that $w = 25\ \text{lb/ft}^2$, $h = 12$ ft, and $L = 28$ ft. Show how loadings are traced through.

3.9. Model the loads on an interior truss of the Institute of Contemporary Art building in Boston, Massachusetts, which is shown in Figure 3.23. Assume a combined dead and live load of 190 lb/ft^2 for the roof and 240 lb/ft^2 for the floor, and ignore the dead weight of the trusses. The building is discussed in more detail in Figure 4.31.

FIGURE 3.22 Example.

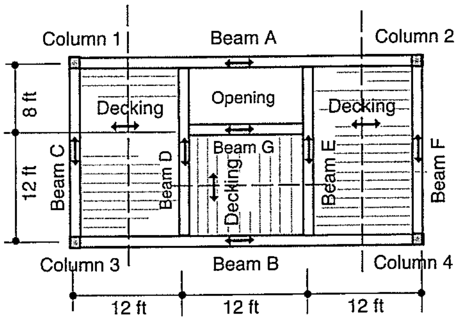


FIGURE 3.23 System diagram for problem no. 3.9. Please also refer to Figure 4.31.