

# CE-370 Safety and Reliability of Engineering Systems

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## SUMMARY OF LECTURE 8 - RELIABILITY OF COMPONENTS IN SERIES

Many engineering systems are arranged such that if any single component fails the complete system will fail. Consider as examples a chain in tension, a pipeline made of individual prefabricated segments, or a one-lane road from point A to B in which if any single portion of the road fails, the road is unable to carry traffic between the two points. These systems are referred to as chain systems or in-series systems. In the context of structural engineering these are referred to as “fracture critical” structures. Mathematically we can express the reliability of such systems as

$$R_s(t) = \prod_{i=1}^n R_i(t) \quad (1)$$

where  $n$  is the number of components in the system and  $R_i(t)$  is the reliability function of component  $i$  at time  $t$ .

Since the reliability of each component can be expressed as

$$R_i(t) = R_{o,i} e^{-\int_0^t r_i(\tau) d\tau} \quad (2)$$

where  $r(t)$  is the failure rate function and  $R_o$  is the initial reliability at  $t = 0$ , then the reliability of the in-series system can be written as

$$R_s(t) = \left( \prod_{i=1}^n R_{o,i} \right) e^{-\int_0^t (\sum_i^n r_i(\tau)) d\tau} \quad (3)$$

For the special case of an in-series system with exponential components with failure rate  $\lambda_i$  we have

$$R_s(t) = \left( \prod_{i=1}^n R_{o,i} \right) e^{-(\sum_i^n \lambda_i)t} \quad (4)$$

### Example - In-Series System

Consider a system with 1,2 and 4 components in-series system. Each component is exponential with failure rate  $\lambda = 1$ . In the figure below we show the reliability function for the various units. As can be seen, as time elapses the reliability of the chain system becomes insignificant with respect to the single unit, specially as the number of components increases. To illustrate this quantitatively consider at  $t = 1$ , the respective reliability of the 1,2 and 4 component system equals 0.37,0.13 and 0.02. Clearly, for the 4-component system to achieve the same level of reliability as the 1-component system the value of  $\lambda$  for each component needs to be 4 times lower.

