CE-370 Reliability of Engineering Systems

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SUMMARY OF LECTURE 1 - BASIC DEFINITIONS

The purpose of this lecture is to establish some basic definitions and concepts that will be relevant to the development of the course.

System: A system is assemblage or combination of interacting parts forming a unit.

Engineered System: An engineered system is defined as an assemblage or combination of multiple parts *designed by engineers* to work together for a common purpose or goal(to satisfy a need of society). Engineering systems are multi-dimensional, interconnected and their relationship with the environment and human operators cannot be neglected.

Model: A model $\mathcal{M}$ is an idealization of the system of interest. Models of engineering systems typically involve a combination of physical and mathematical models. Mathematical models used in engineering analysis will often be a result of selecting from the following characteristics:

- *estimated from data or derived from first principles*
- *numerical or analytical*
- *stochastic or deterministic*
- *micro(nano) or macro*
- *discrete or continuous*
- *finite dimensional or infinite dimensional*
- *qualitative or quantitative*

In addition, issues regarding *model size vs. required predictive accuracy* are also well known and must be studied within the context of each specific application. Below are two quotes that illustrate the struggle between these two issues.

> All models are wrong, some are useful.
>
> G. Box

> The best material model of a cat is another, or preferably, the same cat.
>
> The Role of Models in Science, Philosophy of Science(1945), N. Weiner and A. Rosenblueth

Model Class: A model class $\mathcal{C}$ is a parameterized family of models within a given type of models. Consider as an example the class of all partial differential equations of the form

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

This corresponds to the one-dimensional elastic wave propagation model and the only parameter in the model c represents the wave propagation velocity and can expressed as function of the density (ρ) and elastic modulus (E) of the material as $c = \sqrt{\frac{E}{\rho}}$. This model is continuous in time and space (infinite dimensional), analytical, derived from first principles, deterministic and quantitative.
On the other hand consider an n^{th} order autoregressive model of the form

$$y(k+1) = \sum_{i=0}^{n} a_i y(k-i) + \omega(k)$$

where $y(k) \in \mathbb{R}$ and $a_i \in \mathbb{R}\ \forall i$ and ω is a random process. This model class can be parameterized through the regression coefficients a_i and the properties of the random process. If we have data from a system, we can perform an estimation of the parameters a_i that best fit the data. In this case the model type is stochastic, identified from data, discrete in time, numerical, finite dimensional and quantitative.

Model Parameters: Model parameters θ represent quantities that specify a particular member of the model class $\mathcal{C}$. These quantities are themselves models of characteristics of the system. In the previous example, the wave propagation velocity is a function of the density and elastic modulus. Although one can argue that density (ρ) is an intrinsic physical quantity of the system since it relates to a fundamental concept of the number of molecules in a given volume (Avogrado constant). On the other hand, the elasticity (E) is itself an idealization of the system's capability to deform under the application of loads. *Thus it is important to realize that generally parameters are not a property of the system, but a property of the model.*

Uncertainty: Originates from the fact that we are incapable of reproducing measured system behavior using models, therefore we are uncertain about the accuracy of our model to predict unobservable system behavior. Consider a mechanical system, such as a cantilever subjected to a initial disturbance and we measure its acceleration response at a given point using an accelerometer. If our model predictions do not match the system response (to within the level allowed by measurement noise), we become aware that other unmeasured quantities predicted by our model, such as the stress at a given point of interest, might also be inaccurate. Thus we have *uncertainty* about the stress estimation by our model.

Uncertainty Quantification: Is the process of systematically using discrepancies between model predictions and system observations in order to quantitatively assess the inaccuracy of model predictions regarding unobservable system quantities. One way to achieve this is to assign uncertainty to the model class, model parameters and(or) input variables and use the now "uncertain model" to make stochastic predictions about the system response. However notice that the model parameters are not truly uncertain, they can not be, since they don't exist in reality! Assigning uncertainty to model parameters is not an end in itself, it is simply an intermediate and auxiliary step to quantify the uncertainty in predicting system response quantities using a model.

In some contexts it is important to draw a distinction between the source of our uncertainty. Traditionally the following distinction is made:

- *Epistemic uncertainty*: uncertainties that we could in principle reduce by increasing our knowledge. These might occur because we have not measured a quantity sufficiently accurately, or because our model neglects certain effects, or because particular data are deliberately hidden.
- *Aleatoric uncertainty*: uncertainties that can not be suppressed by more accurate measurements or additional knowledge. Consider for example the effect of human behavior (an operator) in a system's performance. No matter how much information one obtains about that operator, it is impossible to determine what his reaction will be in a given situation in which his/her decision affects the system performance.

Many researchers in the field of statistics (particularly, Bayesian statistics), do not make the above distinction, and attribute all uncertainty to lack of knowledge (epistemic). Although there are strong arguments to support such a stance, the author still claims there are situations, such as those related to human behavior and free will, that impossibilitate increasing our predictive capability based on additional data (not the same as information).

There are various theoretical frameworks to assess uncertainty, some of the most relevant ones are:

- *Probability theory*
- *Fuzzy logic*
- *Evidence theory (Dempster-Shafer)*
- *Possibility theory*

In this course we will restrict our attention to probability theory with some emphasis in measure theory.

Failure: A system has failed when it loses the capability to fulfill at least one of its intended purposes. In the context of structural engineering we refer to failure as the violation of at least one of the various system limit states. Limit states are undesirable structural behaviors linked to undesirable system performance. Some limit states considered in structural design are: partial or complete collapse, excessive deformations or vibrations, cracks exceeding a certain critical width, among others.

Reliability: Is the probability that a component or system will not fail during a specified interval of time.

Risk: The risk induced by a certain event is the product of the probability of occurrence of the event and the consequences of such event.

Risk-Based Decision Making: Is the process of making rational (correct) decisions with the objective of minimizing the total expected risk. The difference between a correct decision and a good decision is the time at which the decision is made with respect to the time at which the decision is evaluated. A good decision can only be evaluated in hindsight, while a correct decision is one in which all available information has been used in order to maximize the objective function.

The main difference between scientists and engineers is that engineers make decisions. Decisions such as the type of foundations used in a suspension bridge, the thickness of a pressurized vessel, the amount and type of chemicals used in a water purification system, the location of a nuclear power plant or launch a spaceship under questionable weather conditions.

In performing risk analysis there are certain basic steps that must be undertaken independently of the system being analysed, these are:

- *Context Definition*: Establish the relationship between the risk analyst, the system, and the internal and external influences.
- *Criteria Definition*: Defining the criteria against which the results of the risk analysis are to be compared. Cultural, humanitarian, social, legal, financial, and technical aspects must be considered.
- *Hazard Identification*: This involves identifying the various ways in which the system might fail and what might occur under adverse circumstances. This process typically involves: (1) define the structure of the system , (2) identify hazard scenarios, and (3) perform a hazard scenario analysis.
- *Risk Analysis*: This involves computing the probability associated with each hazard and their consequences. In this process, components and sub-systems must be analyzed for probability of occurrence and the various probabilities combined.
- *Sensitivity Analysis*: This involves determining the effect of changes in the input variables to the risk estimate. Greater sensitivity of a certain input indicates that greater care is required in defining such inputs accurately.
- *Risk vs. Criteria*: A comparison of the calculated risk with the acceptable risk.
- *Risk Treatment*: For cases in which the estimated risk exceeds the pre-defined criteria, several steps could be considered: (1) risk avoidance (2) risk reduction (3) risk transfer and (4) risk acceptance.

For more details on some of the definitions above please refer to:

1. Stewart, M. and Melchers, R., Probabilistic Risk Assessment of Engineering Systems, 1997, Chapman and Hall.
2. Gershenfeld, N., The Nature of Mathematical Modeling, 1999, Cambridge Press.